A Simple Block Coding Scheme for Peak-to-Average-Power Ratio Reduction in Multi-Carrier Systems

Yoonill Lee
Purdue University Northwest
lee2273@pnw.edu

Abstract

Orthogonal frequency-division multiplexing (OFDM) is an attractive modulation technique for mitigating effects of delay spread in a multipath channel. However, the main disadvantage of OFDM is that it has a large peak-to-average-power ratio (PAPR). One technique for PAPR reduction is Scheme I introduced by Jones et al. [1], which is a block coding scheme using an odd parity check bit and mapping a 3-bit data word onto a 4-bit code word. One disadvantage of Scheme I is that it can only reduce PAPR when the length of the code word is an integer multiple of 4. Another technique is Scheme II introduced by Fragiacomo et al. [2] for longer code words. In Scheme II, where M and N are defined as the length of a code word and the length of a data word, respectively, one additional bit representative of the complement of the (N-1)th bit of the data word is upended as the (N+1)th bit of the data word such that M becomes N+1. Effectively, PAPR can be reduced though M is not an integer multiple of 4, allowing for less-limiting selection of M. However, gains in PAPR reduction are marginal as M increases. This paper proposes a new approach called Scheme III to obtain more PAPR reduction, by adapting Scheme I with two additional bits. Accordingly, M becomes N+2 rather than N+1 as in Scheme II. Scheme III is suited for large numbers of sub-carriers at the expense of two additional bits.

Introduction

A multi-carrier transmission scheme such as orthogonal frequency-division multiplexing (OFDM) has been proposed for many wireless applications because it is an attractive modulation technique for mitigating effects of delay spread in a multi-path channel. However, the main disadvantage of OFDM is that it has a large peak-to-average-power ratio (PAPR), which can result in significant distortion when OFDM signals are passed through a nonlinear device such as a transmitter power amplifier. OFDM can be combined with multiple-input multiple-output (MIMO) technology (e.g., multiple antennas at the transmitter and receiver) to increase the diversity gain to enhance the system capacity. Such MIMO-OFDM system is a key technology for next generation mobile communications as well as wireless local area network (WLAN) [3-5]. The large PAPR problem in OFDM carries over to a MIMO-OFDM system because it is based on OFDM. A number of approaches have been proposed to reduce PAPR for multi-carrier transmission. In general, the PAPR reduction techniques can be classified into the following categories: clipping and filtering, coding, selective mapping, partial transmit sequence, nonlinear companding transforms, tone reservation, tone injection, and scrambling [6-11].

In this paper, however, we only focus on the coding technique category. The first section of this paper explains the definition of PAPR and the peak envelope power for the uncoded data words. The second section discusses Scheme I, Scheme II, and the proposed Scheme III and their corresponding simulation results. The third section provides concluding remarks.

Block Coding Scheme with an Odd Parity Check Bit (Scheme I)

The complex envelope of the transmitted OFDM signal can be written as

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{M-1} d_{n,m} \, e^{j2\pi f_m t} \, g(t-nT) \tag{1}$$

where $d_{n,m}$ is the data symbol for the m-th sub-carrier of the n-th OFDM symbol. M is the number of sub-carriers (or length of code words), f_m is the m-th sub-carrier, T is an OFDM symbol duration, and $g(t)$ is a rectangular pulse shape of duration T. With no loss in generality, we can omit the index n for simplicity. Then eq. (1) can be simply expressed as

$$s(t) = \sum_{m=0}^{M-1} d_m \, e^{j2\pi f_m t} \tag{2}$$

Let $p(t)$ be the envelope power of the OFDM signal. Since $s(t)$ is a complex number, $p(t)$ is $s(t)s^{*}(t)$, where * denotes a conjugate of a complex number. If the power in each sub-carrier is normalized to 1 watt, then the average power is M watts.
The PAPR Γ (in dB) is defined as

$$\begin{aligned}\Gamma &= 10\log_{10}\left\{\frac{peak\ envelope\ power}{average\ power}\right\} \\ &= 10\log_{10}\left\{\frac{peak\ p(t)}{M}\right\}\end{aligned} \tag{3}$$

First, we discuss the uncoded data words and then explain how the block coding scheme using an odd parity check bit works for four sub-carriers (M = 4). The peak envelope powers for all possible uncoded data words are shown in Table 1. Let us define PEP as the peak envelope power of *p(t)*. The first PEP is 16.00 watts when the data words are 0000, 0101, 1010, and 1111. The second PEP is 9.44 watts when the data words are 0011, 0110, 1001, and 1100. All other PEPs are the same values of 7.07 watts. Thus if we can remove data words that generate large PEPs such as 16.00 watts and 9.44 watts, we can reduce the PAPR. This is the main motivation behind block coding.

Table 1. PEP for all possible uncoded data words (N = 4)

Data words	d_1	d_2	d_3	d_4	PEP (watt)
0	0	0	0	0	16.00
1	0	0	0	1	7.07
2	0	0	1	0	7.07
3	0	0	1	1	9.44
4	0	1	0	0	7.07
5	0	1	0	1	16.00
6	0	1	1	0	9.44
7	0	1	1	1	7.07
8	1	0	0	0	7.07
9	1	0	0	1	9.44
10	1	0	1	0	16.00
11	1	0	1	1	7.07
12	1	1	0	0	9.44
13	1	1	0	1	7.07
14	1	1	1	0	7.07
15	1	1	1	1	16.00

Jones et al. introduced the block coding scheme using an odd parity check bit [1]. The basic idea is that a 3-bit data word is mapped onto a 4-bit code word using an odd parity check bit. The first three bits c_1, c_2, and c_3 in the code word are the same as d_1, d_2, and d_3 in the data word. However, the fourth bit c_4 in the code word is an odd parity check bit. For example, if a 4-bit data word is $d_1 = 0$, $d_2 = 1$, $d_3 =0$, and $d_4 = 1$, then the 4-bit code word using an odd parity check bit is $c_1 = 0$, $c_2 = 1$, $c_3 = 0$, and $c_4 = 0$. Table 2 shows the peak envelope powers for all possible code words using an odd parity check bit. As the table indicates, the peak envelope powers have the same value (i.e., 7.07 watts) because the first and second PEPs are removed. The results of PAPR with different lengths of code words for uncoded data words and Scheme I for comparison appear in Table 3: Scheme I works well when the length of code words is an integer multiple of 4. The PAPR value is 2.48 dB in the case of M = 4. Thus, the amount of PAPR reduction (ΔdB_I) relative to the uncoded data words is as much as 3.54 dB. However, it has the same PAPR as the uncoded data words if the length of code words is not an integer multiple of 4. This is the main drawback of Scheme I. Block coding Scheme II, explained in the next section, addresses how to solve this problem.

Table 2. PEP for all possible code words using scheme I (M = 4)

Code words	c_1	c_2	c_3	c_4	PEP (watt)
0	0	0	0	1	7.07
1	0	0	0	1	7.07
2	0	0	1	0	7.07
3	0	0	1	0	7.07
4	0	1	0	0	7.07
5	0	1	0	0	7.07
6	0	1	1	1	7.07
7	0	1	1	1	7.07
8	1	0	0	0	7.07
9	1	0	0	0	7.07
10	1	0	1	1	7.07
11	1	0	1	1	7.07
12	1	1	0	1	7.07
13	1	1	0	1	7.07
14	1	1	1	0	7.07
15	1	1	1	0	7.07

Table 3. PAPR (in dB) comparisons for uncoded data words and Scheme I

Length of code words (M)	Uncoded	Scheme I	ΔdB_I
4	6.02	2.48	3.54
5	6.99	6.99	0
6	7.78	7.78	0
7	8.45	8.45	0
8	9.03	6.53	2.50
9	9.54	9.54	0
10	10.00	10.00	0
11	10.41	10.41	0
12	10.79	9.21	1.58
13	11.14	11.14	0
14	11.46	11.46	0
15	11.76	11.76	0
16	12.04	10.88	1.16

Block Coding Scheme with One Additional Bit (Scheme II)

As shown in the previous section, the disadvantage of block coding using an odd parity check (Scheme I) is that it can only reduce PAPR when the length of the code words is an integer multiple of 4. To be specific, it has the smallest PAPR value with M = 4. When the length of code words is not an integer multiple of 4, it does not eliminate code words that generate large powers. Fragiacomo et al. introduced another approach for longer code words [2]. Let M and N be defined as the length of a code word and the length of a data word, respectively. Instead of using an odd parity check bit, one additional bit representative of the complement of the (N-1)th bit of the data word is upended as the (N+1)th bit of the data word such that M becomes N+1. Table 4 shows the PEP for all possible code words using one additional bit with M = 5. The first PEP is 13.33 watts and the second PEP is 13.00 watts. For comparison, Table 5 shows the PAPR for different lengths of code words for uncoded data words and Scheme II. The PAPR values are reduced even though the length of code words is not an integer multiple of 4. Thus Scheme II has a more general way to select the length of code words compared to Scheme I. However, the amount of PAPR reduction (ΔdB_{II}) does not improve much when the length of code words increases. To obtain better PAPR reduction for large code words, we propose a new approach by adapting Scheme I with two additional bits.

Table 4. PEP for all possible code words using Scheme II (M = 5)

Code words	c_1	c_2	c_3	c_4	c_5	PEP (watt)
0	0	0	0	0	1	10.51
1	0	0	0	1	1	13.33
2	0	0	1	0	0	10.48
3	0	0	1	1	0	13.00
4	0	1	0	0	1	13.33
5	0	1	0	1	1	10.51
6	0	1	1	0	0	13.00
7	0	1	1	1	0	10.48
8	1	0	0	0	1	10.48
9	1	0	0	1	1	13.00
10	1	0	1	0	0	10.51
11	1	0	1	1	0	13.33
12	1	1	0	0	1	13.00
13	1	1	0	1	1	10.48
14	1	1	1	0	0	13.33
15	1	1	1	1	0	10.51

Table 5. PAPR (in dB) comparisons for uncoded data words and Scheme II

Length of code words (M)	Uncoded	Scheme II	ΔdB_{II}
4	6.02	3.73	2.29
5	6.99	4.26	2.73
6	7.78	5.09	2.69
7	8.45	5.58	2.87
8	9.03	6.53	2.50
9	9.54	7.36	2.18
10	10.00	8.06	1.94
11	10.41	8.67	1.74
12	10.79	9.21	1.58
13	11.14	9.69	1.45
14	11.46	10.12	1.34
15	11.76	10.52	1.24
16	12.04	10.88	1.16

Proposed Coding Scheme Adapting Scheme I with Two Additional Bits (Scheme III)

In this section, we propose a novel block coding algorithm, as follows:

Step 1: Apply Scheme I to the data words
Step 2: Upend the complement of the (N-1)th bit of the code word as the (N+1)th bit of the code word
Step 3: Upend the complement of the odd parity check bit as the (N+2)th bit of the code word

Accordingly, M becomes N+2 rather than N+1 as in Scheme II. Table 6 shows the peak envelope power with different lengths of code words for proposed Scheme III. The first PEP is 16.98 watts and the second PEP is 13.29 watts. Although these values are larger than those in Scheme I and II, the PAPR values are smaller compared to the two previous schemes. Moreover, the amount of PAPR reduction (ΔdB_{III}) is much larger than the two previous schemes when the length of code words increases. Thus Scheme III is suited for large numbers of sub-carriers at the expense of two additional bits. A comparison for all schemes with different lengths of code words is shown in Table 7.

Table 6. PEP for all possible code words using Scheme III (M = 6)

Code words	c_1	c_2	c_3	c_4	c_5	c_6	PEP (watt)
0	0	0	0	1	1	0	13.29
1	0	0	0	1	1	0	13.29
2	0	0	1	0	0	1	16.98
3	0	0	1	0	0	1	16.98
4	0	1	0	0	1	1	13.29
5	0	1	0	0	1	1	13.29
6	0	1	1	1	0	0	16.98
7	0	1	1	1	0	0	16.98
8	1	0	0	0	1	1	16.98
9	1	0	0	0	1	1	16.98
10	1	0	1	1	0	0	13.29
11	1	0	1	1	0	0	13.29
12	1	1	0	1	1	0	16.98
13	1	1	0	1	1	0	16.98
14	1	1	1	0	0	1	13.29
15	1	1	1	0	0	1	13.29

Table 7. PAPR (in dB) comparisons for all schemes

M	Uncoded	Scheme I	ΔdB_{I}	Scheme II	ΔdB_{II}	Scheme III	ΔdB_{III}
4	6.02	2.48	3.54	3.73	2.29	3.73	2.29
5	6.99	6.99	0	4.26	2.73	4.26	2.73
6	7.78	7.78	0	5.09	2.69	4.52	3.26
7	8.45	8.45	0	5.58	2.87	5.58	2.87
8	9.03	6.53	2.50	6.53	2.50	6.02	3.01
9	9.54	9.54	0	7.36	2.18	6.59	2.95
10	10.00	10.00	0	8.06	1.94	6.64	3.36
11	10.41	10.41	0	8.67	1.74	7.44	2.97
12	10.79	9.21	1.58	9.21	1.58	7.78	3.01
13	11.14	11.14	0	9.69	1.45	8.15	2.99
14	11.46	11.46	0	10.12	1.34	8.54	2.92
15	11.76	11.76	0	10.52	1.24	9.07	2.69
16	12.04	10.88	1.16	10.88	1.16	9.54	2.50

Simulation Results

In this section we analyze all the schemes previously discussed. Figures 1a and b show the envelope powers for uncoded data words and under Scheme I, respectively. Figures 2a and b show the PAPR values for uncoded data words and under Scheme II, respectively. Similarly, Figures 3a and b show the PAPR for uncoded data words and under Scheme III, respectively. Finally, Figures 4a and b show the PAPR comparison and reduction for all the schemes, respectively. As shown, the proposed Scheme III shows improved performance compared to those of the two previous schemes when the length of code words increases.

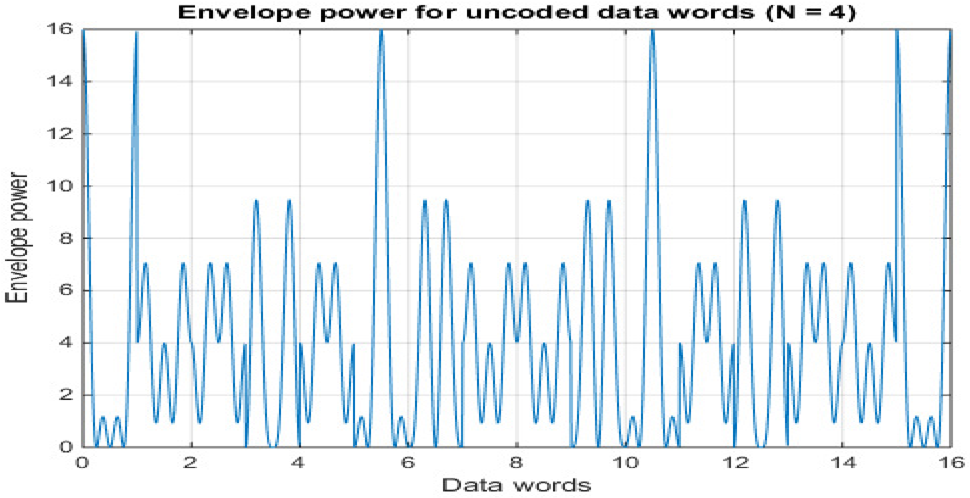

Figure 1a. Envelope power for uncoded data words

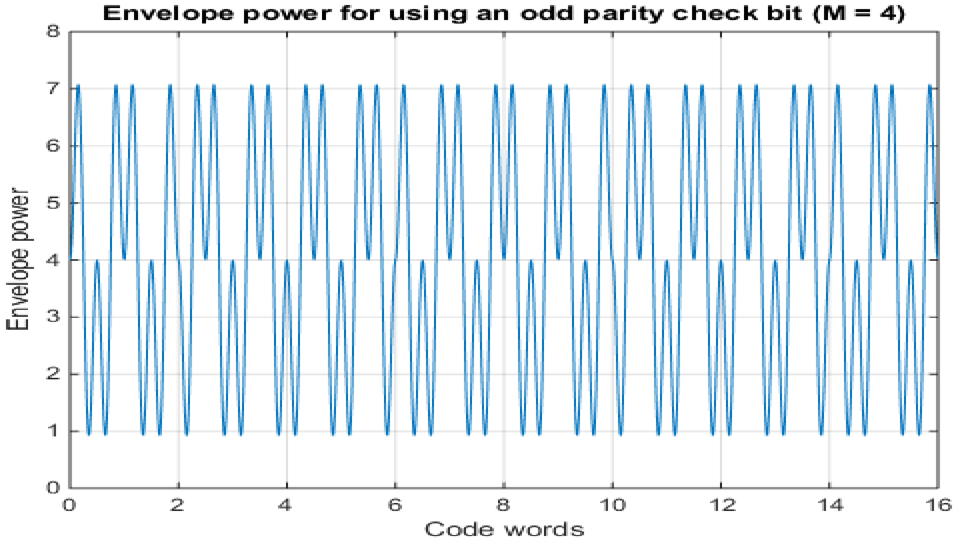

Figure 1b. Envelope power using Scheme I

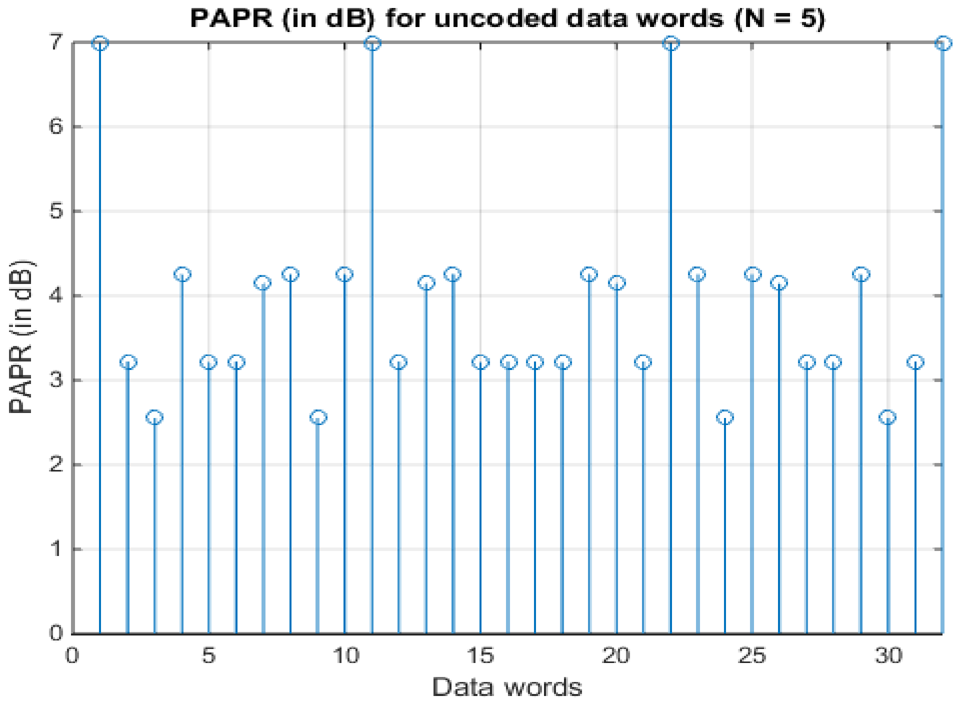

Figure 2a. PAPR (in dB) for uncoded data words

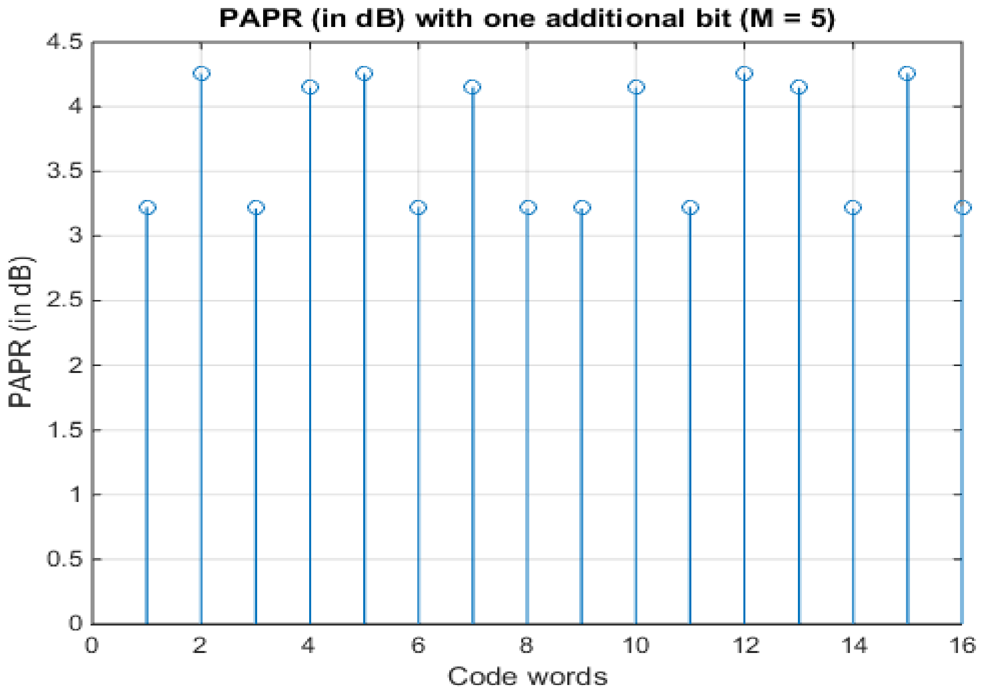

Figure 2b. PAPR (in dB) using Scheme II

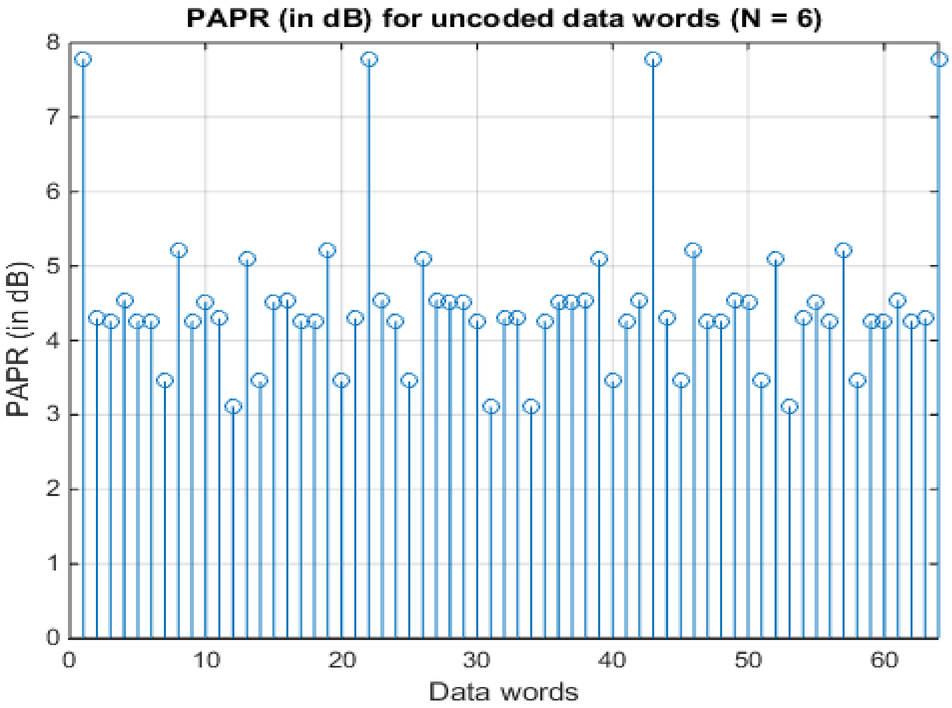

Figure 3a. PAPR (in dB) for uncoded data words

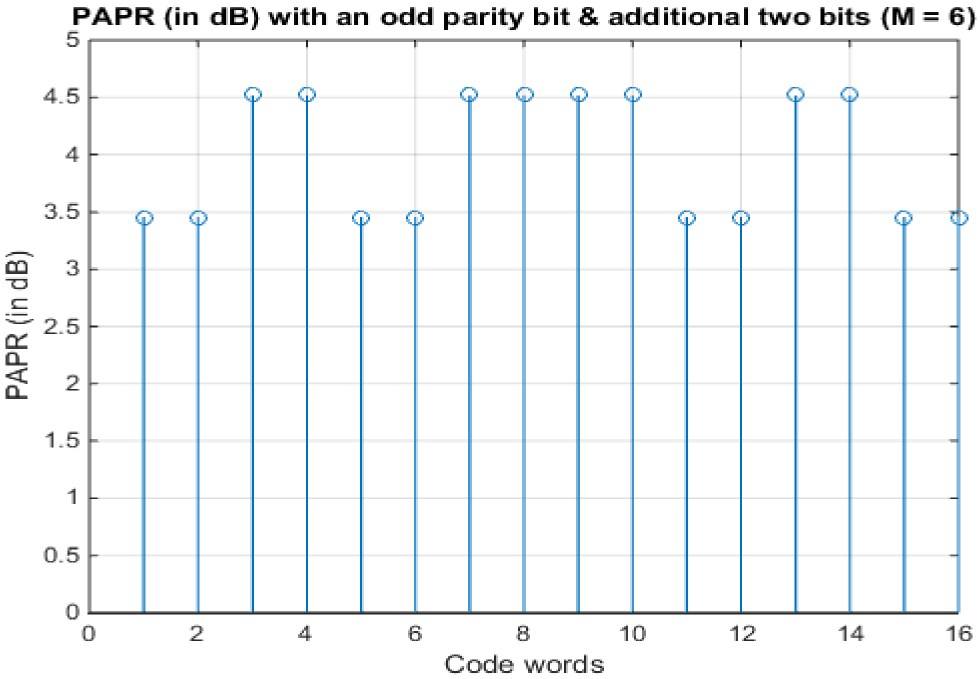

Figure 3b. PAPR (in dB) using Scheme III

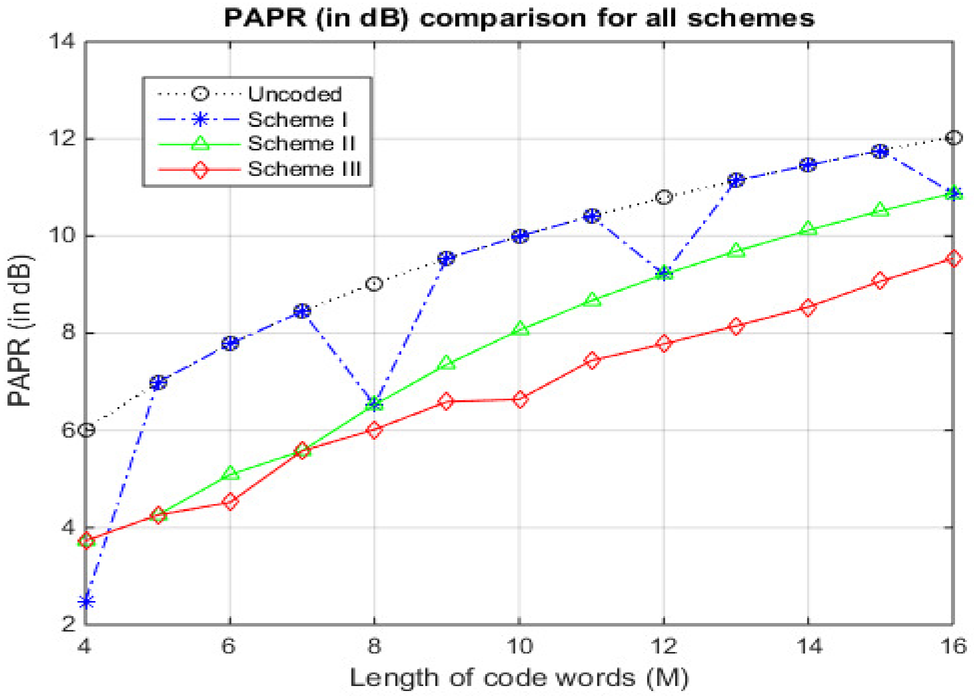

Figure 4a. PAPR (in dB) comparison for all schemes

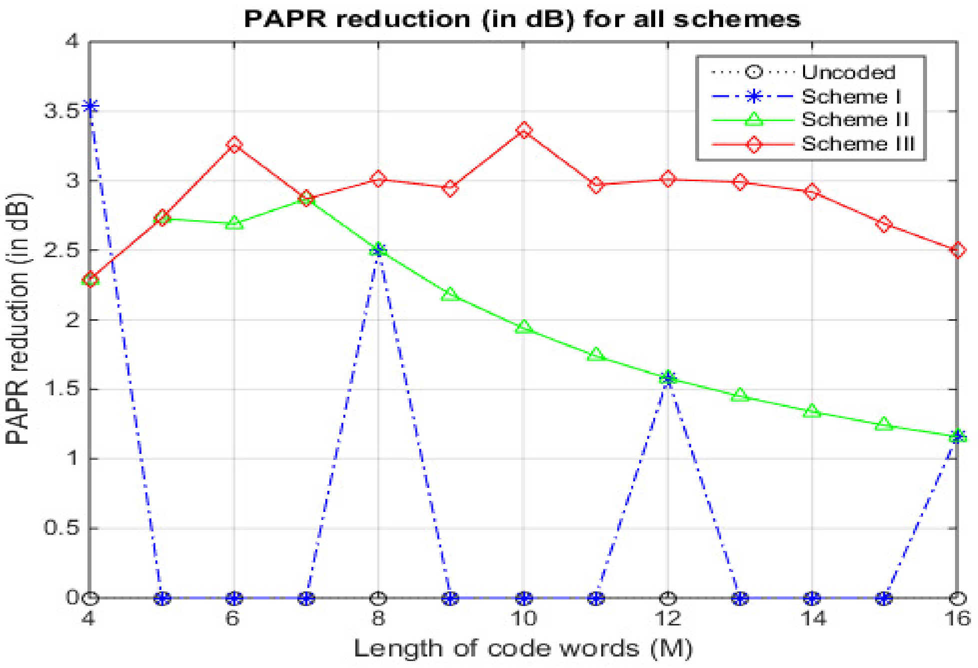

Figure 4b. PAPR reduction (in dB) for all schemes

Conclusion

As a PAPR reduction scheme, we have proposed a new block coding Scheme III by adapting Scheme I with two additional bits. Scheme I only reduces PAPR when the length of code word is an integer multiple of 4. This is the main drawback of Scheme I. Scheme II reduces PAPR even though the length of code word is not an integer multiple of 4. However, the PAPR reduction gains are marginal when the length of code word increases. Scheme III shows improved performance compared to those of the two previous schemes when the length of code words increases. For example, the amount of PAPR reduction with M = 8 for Scheme I, Scheme II, and Scheme III are 2.50 dB, 2.50 dB, and 3.01 dB, respectively. For M = 16, they are 1.16 dB, 1.16 dB, and 2.50 dB, respectively. As a result, Scheme III is suited for a large numbers of sub-carriers at the expense of two additional bits.

References

[1] Jones, A. E., Wilkinson, T. A., & Barton, S. K. (1994). Block coding scheme for reduction of peak to mean envelope power ratio of multi-carrier transmission schemes. *Electronics Letters*, 2098-2099.

[2] Fragiacomo, S., Matrakidis, C., & O'Reilly, J. J. (1998). Multi-carrier transmission peak to average power reduction using simple block code. *Electronics Letters*, 953-954.

[3] Stuber, G. L., J. R. Barry, J. R., Mclaughlin, S. W., Li, Y, Ingram, M. A., & Pratt, T. G. (2004). Broadband MIMO-OFDM wireless communications. *Proceedings of the IEEE*, *92*(2), 271-294.

[4] Iqbal, Z., Nooshabadi, S., & Lee, H. N. (2012). Analysis and design of coding and interleaving in a MIMO-OFDM communication system. *IEEE Transactions on Consumer Electronics*, *58*(3), 758-766.

[5] Cho, Y. S., Kim, J. W., Yang, W. Y., & Kang, C. G. (2013). *MIMO-OFDM wireless communications with MATLAB*. Singapore: John Wiley & Sons (Asia).

[6] Han, S. H., & Lee, J. H. (2005). An overview of peak-to-average-power ratio reduction techniques for multicarrier transmission. *IEEE Wireless Communications*, *12*(2), 56-65.

[7] Jiang, T., & and Wu, Y. (2008). An overview: Peak-to-average power ratio reduction techniques for OFDM Signals. *IEEE Transactions on Broadcasting*, 54(2), 257-268.

[8] Neem R. V. (1996). OFDM codes for peak-to-average-power reduction and error correction. In *Proceedings of IEEE GLOBECOM* (pp. 740-744). London, UK.

[9] Tellambura, C. (1997). Use of M-sequences for OFDM peak-to-average-power ratio reduction. *Electronics Letters*, 1300-1301.

[10] Davis, J. A., & Jedwab, J. (1999). Peak to mean power control in OFDM, Golay complementary sequences, and Reed-Muller codes. *IEEE Transactions on Information Theory*, *45*(7), 2397-2417.

[11] Zhang, Y., Yongacoglu, A., Chouinard, J. Y., & Zhang, L. (1999). OFDM peak power reduction by sub-block coding and its extended versions. In *Proceedings of the IEEE Vehicular Technology Conference* (pp. 695-699). Houston, TX.

Biography

YOONILL LEE is currently an assistant professor of Electrical Engineering Technology at the Purdue University Northwest. He earned his BS and MS in Electrical Engineering from Yonsei University in Seoul, South Korea, in 1984, 1986, respectively, and MS in Electrical Engineering from Oklahoma State University in Stillwater, OK, in 1992, and PhD in Electrical Engineering from the University of South Florida in Tampa, FL, in 2001. He worked at Cisco Systems as a communications engineer. His research interests include CDMA, multi-carrier systems, and MIMO technology. Dr. Lee can be reached at lee2273@pnw.edu.