Strategic Supply Chain Source Selection using Bayesian Multi Criteria Decision Based Approach

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Abstract

Companies increasingly confront with finding potential sources to buy materials or subcontract items for processing its primary outputs due to global competition and short life cycles of products. This study proposes an integrated approach that combines traditional multi-attribute decision analysis (MADA) method and Bayesian technique to determine the best choice among all possible choices of supply sources to ensure the best values. The order preference by similarity to ideal solution (TOPSIS) algorithm is a preferred MADA approach used here to determine the most desirable supply source assessing both tangible and intangible attributes of benefits and costs among all the alternatives. TOPSIS approach obtains the attribute weights given by the decision makers. However, any influence or biases on weights or the lack of information may result in misleading priority orders of alternatives. The complexity increase as the precise scoring of attributes become challenging due to data uncertainty, incomplete and non-obtainable information and constant change of preferences on attributes. The aim is to determine the best supply source assessing both tangible and intangible attributes of benefits, changing performances and costs among alternatives.

The proposed approach uses TOPSIS framework utilizing two weight coefficient approaches: (i) Entropy technique, (ii) Bayesian technique integrating past information on performance ratings of alternatives and decision makers' current view regarding the attribute weights. The Bayesian approach enables the updated (posterior) weights of attribute in the selection process enumerating new evidences that decision makers may need make a significant change in the performance rating of an alternate. A numerical example with five attributes (e.g., price, delay, mean time between failure, alliance to the company, commercial terms, and compliance to specifications) and ten supply alternatives is presented.

Introduction

Companies increasingly procure input materials from outside suppliers, create values by the processing inputs into its primary outputs before sending to the customers. Major input

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components include raw material, semi-finished components, subassemblies, tools, spare parts, office stationary, etc. As competition intensifies, the input source selection from the suppliers has become a vital decision since it involves the material costs, quality and reliability of the product, customer satisfaction, as well as the company's overhead cost. In many real life situations, it is essential to consider multiple attributes of a deal in addition to price for negotiation and selection through a more effective information exchange of buyer's preferences and supplier's offerings [1]. Companies need to develop various supplier selection strategies and use the resources at their disposal to find qualified suppliers [2]. Companies use e-sourcing, auction, reverse auction, and tenders to purchase specific goods and services from the supply sources. E-sourcing is a process referred for identifying, evaluating and configuring the optimal grouping of buyers and suppliers in a supply chain that respond to changing market demands [3].

The multi-attribute decision analysis (MADA) approach finds the potential source from a finite number of feasible alternatives based on few selected quantitative and qualitative attributes. Numerous researchers put forward some theories and methods to focus on the application of MADA in a variety of areas to explore the issue of multi-attribute decision making. TOPSIS was developed as a classical MADA method to find the best option from all of the feasible alternatives [4, 5]. It has been successfully adopted in various fields, e.g., intelligent information, location analysis, construction processes, human resources management, transportation, product design, manufacturing, water management and quality control, etc.

The evaluation of weights is critical in TOPSIS method since there are uncertainties involve in attribute ratings. The entropy technique uses quantitative data conveyed to the ratings of each alternative in regards to criteria to avoid the subjectivity in determining weights. The advantages of entropy integrated TOPSIS method include straightforward computational process and avoids uncertain human judgment on choosing the weight of criteria.

However supplier selection is driven by external uncertainties, such as firm-specific uncertainty, market uncertainty, and behavior uncertainty, which are the difficulties companies have in predicting their future [6]. Information about decision alternatives is often incomplete because of e-sourcing documents, lack of data, intangible characteristics and limitations on information processing capabilities. The subjectivity of assigning weights to attributes and incomplete information on data affect the accuracy of the value of weights and final rank orders of the alternatives. The study also considers TOPSIS integrated with Bayesian technique in regards to determining the weight on attributes while considering any new evidence of changes in the performance rating of the candidate sources. For a practical MADA problem, the method to integrate data and experts' subjective choices in the Bayesian process improve the decision-making practice. In general, information are obtained from the past sources, however, decision makers may want to adjust the performance rating if there any ambiguous claim, inadequacy, conflicting information. The advantage of Bayesian technique includes the integration of past information on attribute data and decisional makers' current position surrounding any new information and changes to predict the forthcoming performances. The study directs to solve two specific research questions:

A. Are the rank and selection of alternate sources in MADA process significantly different in regards to the weights implemented in the criteria: (i) Entropy weight technique derived from the attribute data and (ii) Bayesian weights integrated with entropy weights and experts prior information?

B. What is the appropriate modular approach for the decision makers to provide reliable, rational and transparent operational solution for potential source selection?

The rest of the chapter is organized as follows: Section 2 presents the MADA models and theoretical background of TOPSIS algorithm. Section 3 deals with the applicability of the proposed methodology utilizing weight coefficient techniques: (i) Entropy technique to select the weights and (ii) Bayesian weights integrated with entropy weights and the experts' prior information regarding the weights. Section 4 is the numerical illustration of TOPSIS algorithm using Entropy weights and Bayesian weights. The priority rank orders of all feasible alternatives are presented on both the weight coefficient approaches. Section 5 presents the conclusion and suggestions for future work.

TOPSIS Framework

Several researchers improve TOPSIS method to accommodate uncertainly and incomplete information of alternate performance ratings prior to determine the best candidate. The basic principle of the TOPSIS method is that the alternative is chosen using two reference points: the 'shortest distance' from the positive ideal solution and the 'farthest distance' from the negative ideal solution. The concept of TOPSIS to solve multi-objective nonlinear programming problems is an extended technique [5]. MADA model also extended using interval data [6]. Extension of the TOPSIS method for decision-making problems with fuzzy data, where fuzzy number is calculated using the concept of *α-cuts* [7]. A new methodology is developed for solving multi-attribute group decision-making problems using *intuitionistic fuzzy set* (IFS) in which two auxiliary fractional programming models are derived from the TOPSIS to determine the relative closeness coefficient intervals of alternatives [8]. A study on uncertain information and aggregate the multi-period evaluations used dynamic multi-attribute decision making integrated with the concepts of grey number and Minkowski distance function [9]. The mathematical expression of the MADA model is defined by a set of alternatives and attributes. Alternatives are denoted by $B = \{B_1, B_2, \ldots, B_m\}$, from which decision maker selects the optimal alternative, rendering an identified set of criteria, indicated by $C = \{C_1, C_2, \ldots, C_n\}$. The procedure to determine ranks among the alternatives using TOPSIS algorithm follows a series of steps [4].

Step 1: Identify evaluation criteria to construct decision matrix:
In this step, the data is expressed in (m×n) matrix with m alternatives and n selection criteria representing the discrete choice between the criteria and alternatives. A MADA method expressed using the following matrix format:

Bids\Criteria	C_1	C_2	. . .	C_n
B_1	$\tilde{x}_{11}$	$\tilde{x}_{12}$	. . .	$\tilde{x}_{1n}$
B_2	$\tilde{x}_{21}$	$\tilde{x}_{22}$	. . .	$\tilde{x}_{2n}$
B_3	$\tilde{x}_{13}$	$\tilde{x}_{32}$	. . .	$\tilde{x}_{3n}$
□	□	□	□	□
B_m	$\tilde{x}_{m1}$	$\tilde{x}_{m2}$	. . .	$\tilde{x}_{mn}$

Matrix element, $\tilde{x}_{ij}$ is the performance rating of alternative B_i corresponds to criteria C_j. The normalization of the data eliminates the differences of and inconsistent scale. The normalized value P_{ij} is calculated as

$$N_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{m} x_{ij}^2}, \qquad i \epsilon (1, 2 \dots, m) \quad j \epsilon (1, 2 \dots, n) \tag{1}$$

where N_{ij} represents the numerical evaluation of alternative B_i correspond to criterion C_j.

Step 2: Construct weighted normalized decision matrix:

The decision matrix elements are updated by multiplying the weights of the attributes. In this study, two forms of weights are implemented: (i) Entropy weights and (ii) Bayesian weights integrated with Entropy weights and experts prior information with the normalized value N (obtained in Step 1) as the following:

$$\widetilde{V_{ij}} = \left(\widetilde{N_{ij}} \times \widetilde{w_j}\right) \tag{2}$$

where $\widetilde{w_j}$ is weights, $\widetilde{V_{ij}}$ is the element in decision matrix, $i \epsilon (1, 2 \dots, m)$; $j \epsilon (1, 2 \dots, n)$.

Step 3: Positive ideal solution and negative ideal solution:

The step is to set of (positive and negative) ideal solution using the benefit criteria, J_b ($J \in J_b$); and cost criteria J_c ($J \in J_c$). The positive ideal reference point, (V^+) and the negative ideal reference point (V^-) are obtained as the following.

$$V^+ = \left\{\left(max_i V_{ij} \middle| J \in J_b\right), \left(min_i V_{ij} \middle| J \in J_c\right)\right\} = \{V_1^+, V_2^+, \dots, V_n^+\} \tag{3a}$$

$$V^- = \left\{\left(min_i V_{ij} \middle| J \in J_c\right), \left(max_i V_{ij} \middle| J \in J_c\right)\right\} = \{V_1^-, V_2^-, \dots, V_n^-\} \tag{3b}$$

Step 4: Separation of measures:

The distance of each alternative from the positive ideal reference point, S_i^+ and negative ideal reference point, S_i^- obtained as follows:

Positive Ideal Separation: $$S_i^+ = \sqrt{\sum_{j=1}^{n} d\left(V_{ij} - V_j^+\right)^2} \quad i = 1, 2, \dots, m \tag{4a}$$

Negative Ideal Separation: $$S_i^- = \sqrt{\sum_{j=1}^{n} d\left(V_{ij} - V_j^-\right)^2} \quad i = 1, 2, \dots, m \tag{4b}$$

where $d\left(V_{ij} - V_j^+\right)$ is the distance between an element and the maximum (or minimum) value.

Step 5: Obtain the closeness co-efficient and rank the alternatives

The relative closeness index R_i is calculated using S_i^+ and Si^-. The R_i value determines the priority rank orders by the following:

$$R_i = \frac{S_i^-}{S_i^+ + S_i^-} \quad i = 1, 2, \dots, m;\ 0 \le R_i \le 1. \tag{5}$$

Closeness index R_i approaching 1 refers to high priority (i.e., close to the positive ideal reference point and away from negative ideal reference point).

Weight Selection on Attributes

The multi-attribute decision analysis models have been widely applied due to easier formation of criteria based alternate selection matrix, direct solution and user-friendly application technique. There are several recent studies that extend TOPSIS methods: TOPSIS analysis integrated with entropy weight techniques and Bayesian networks in TOPSIS to improve the selection procedure in terms of uncertainty.

Entropy Weight Method

In typical MADA method, weights of decision on criteria and the performance rating of the alternative are overly judgmental. However, Entropy weight coefficient technique minimizes the uncertainty and subjective judgments. The method determines the weight using the quantitative information conveyed to rate each alternative [10]. The entropy method is proposed to determine the weight of each alternative depending on the criteria in the data matrix [11]. In the Entropy method, the weight factor is a direct function of the intrinsic value presented in the data. Entropy weight coefficient method to determine weight $\widetilde{w}_j$ for each criteria, C_j ($j = 1, 2, \dots, n$) is the following. Using normalized decision matrix, N_{ij}, and weight coefficient E_j is calculated as follows:

$$E_j = -k \sum_{j=1}^{n} N_{ij} \ln N_{ij} \tag{6}$$

where k (constant) = 1 / (ln (m). The principle of entropy method refers that a criterion tends to be more important, if a greater dispersion is observed in the evaluations of the alternatives. The higher D_j value indicates the importance of the attribute in the decision matrix. The measurement of dispersion D_j for a criterion is calculated as the following:

$$D_j = 1 - E_j. \tag{7}$$

The weight W_j for each attribute C_j is calculated by using the following formula

$$w_j = \frac{D_j}{\sum_{k=1}^{n} D_k} \tag{8}$$

$w_j = \widetilde{w}_1, \widetilde{w}_2, \dots, \widetilde{w}_n$, where $\widetilde{w}_j$ is the weight of jth criterion Cj.

In Entropy approach the attribute weights automatically calculate without direct involvement of the decision makers. Insights obtained using this approach provides realistic measures to compare alternatives.

Bayesian Weight Selection Process

A Bayesian technique is a statistical process model, proven to be efficient for solving MADA model under uncertainty, i.e., models where the qualitative and quantity values of the attribute may change over time. A Bayesian network (BN) is a directed acyclic graphical (DAG) representation of the joint probability distribution to describe the combined set of variables which each variable has a finite set of mutually exclusive states [11]. In Bayesian network, the node represents variables and arcs represents direct connection between nodes. The variables (nodes) are the factors of interest, represented by conditional probability distributions, updates in the Bayesian process as the new information becomes available.

In the procedure the Entropy based technique and Bayesian technique are independent of each other. Bayesian technique utilizing the TOPSIS algorithm is shown in Figure 1.

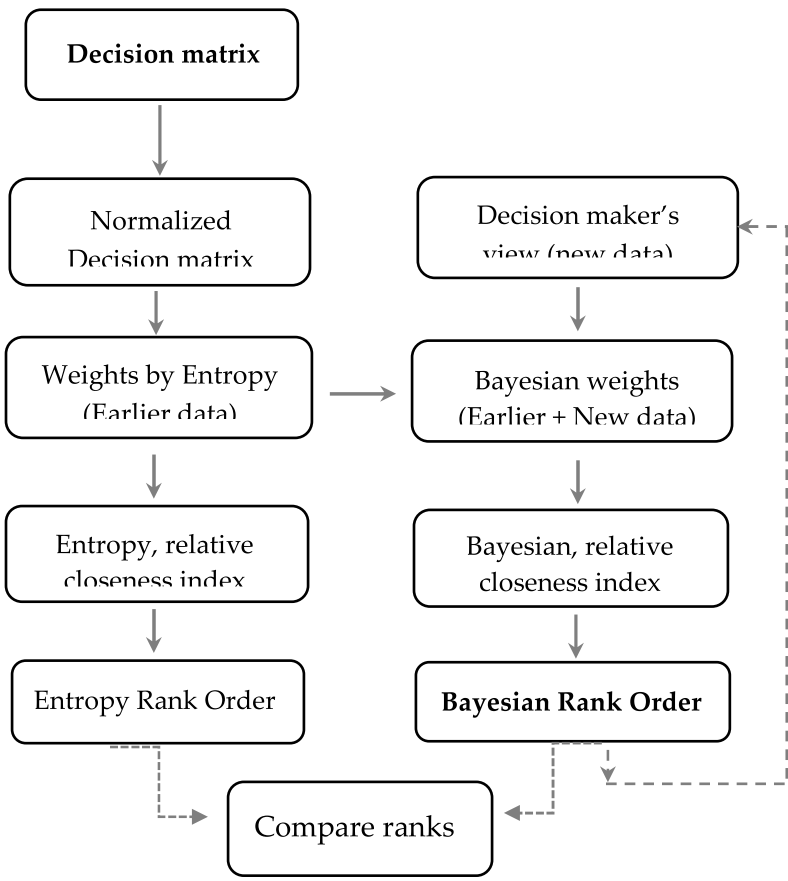

Figure 1: A Bayesian technique integrated in TOPSIS algorithm

In real life the decision makers may not be correct in scoring the weights of criteria as it depends upon the uncertainty of human judgement. In complex decision making problems it is prudent to consider the current and past information of each attribute and the alternatives

as described in the Bayesian process. The priority rank for each alternative can be obtained using both weight factors by the multi-criterial based TOPSIS method.
Bayesian network inferences about the weight of a criteria can be directly used in decision-making tasks. A simple Bayesian network relationship of the prior information of weights assumed by experts and weights obtained using the entropy methods from the quantitative and qualitative data is proposed to estimate the weights of the attributes. The distribution of the updated weight factors of the attributes is shown in Figure 2.

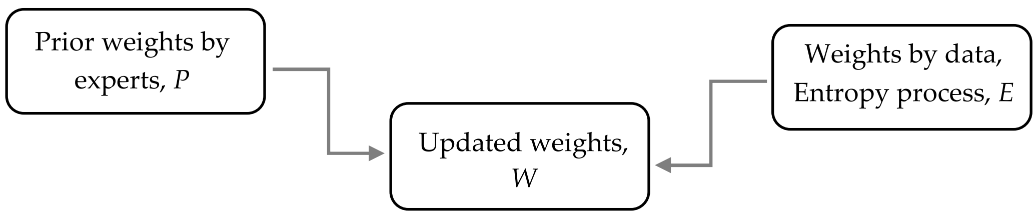

Figure 2: Bayesian Network to estimate the distribution of updated weights

In Bayesian networks, the causal structure and the numerical values can be defined through two different approaches. These are (i) information learned from a dataset and (ii) judgement from experts. The prior weight information P by the experts' presented by the distribution $f(P)$, the data driven weights derived by Entropy method E is presented by the distribution $f(E)$. The probability $f(P, E|W)$ is the updated weight W, given that information of P, E have been obtained (conditional). The scoring of the attribute is based on the assumption that the decision maker uses the prior knowledge about the importance of attributes in the final selection process. The weights using the conditional probability is updated as:

$$f(w|P,E) = \frac{f(w)f(P,E|w)}{\int f(w)f(P,E|w)dW} \tag{9}$$

Empirical Analysis

With the globalization of the market, companies are subjected to procure major supplies from external sources or execute outsourcing to the external organizations electronically. These input components include raw material, semi-finished components, subassemblies, tools, and spare parts, office stationary or part of the manufacturing job to process and complete the company's primary outputs before delivering to the customers. The practice of internet/internet-based supply source, bidding process and multicriteria decision support system to analysis the appropriate bidder are gradually increasing in business-to-business commercial transactions. A study for volume discount cost functions and business constraints model considered e-sourcing of multiple unit of a single item with multi-attributes [12]. With the emergence of internet and growth of the information technology many firms have realized the possibilities for cost savings and increasing their efficiency by using online procurement [13]. The concept of linguistic variables is useful in dealing with situations that are too complex or too poorly defined to be reasonably described in conventional quantitative expressions [14]. A TOPSIS based model for solving the sealed-bid multiple attribute reverse auction problem is proposed to determine the winning bidder while satisfy the interests of both the auctioneer and the bidders [15]. The proposed model is demonstrated using a partial dataset for a seal-bid reserve auction collected from [13]. The data series is the case study to determine the winning bidder comparing 10 closed bid alternatives $B = \{B_1, B_2, \ldots, B_{10}\}$ with respect to five attributes by $C = \{C_1, C_2, \ldots, C_5\}$ presented in Table 1.

Table 1: Decision matrix of 10 alternatives and five attributes

Bids	Price (Euro)	Delay (Days)	Mean Time between Failure	Alliance to company commercial terms	Compliance to specifications
	C_1	C_2	C_3	C_4	C_5
B_1	125.95	1	200000	F	MP
B_2	147.37	3	250000	F	MP
B_3	146.38	4	166667	G	P
B_4	129.66	2	142857	F	F
B_5	151.46	2	250000	MP	F
B_6	180.25	1	250000	MG	G
B_7	168	2	500000	P	F
B_8	125.59	4	111111	P	MG
B_9	125.85	6	83333	F	MP
B_{10}	176.15	2	500000	F	P

The first three attributes C_1, C_2, C_3 are quantitative, i.e., Price, Delay and Mean Time between failure (MTBF), respectively. The next two attributes C_4, C_5, are qualitative in nature, considered as linguistic variables, directly perceived from the bid submitted by the supplier. The weights of C_4 and C_5 are expressed in positive crisp values. In order to determine the performance rating using linguistic variables, the bids are rated on the basis of level of their compliance to the components or sub-attributes [13]. Linguistic variable scoring of the attributes (Alliance to company commercial terms and Compliance to specifications) may be provided by the decision makers (bidders or auctioneer). For simplicity, the linguistic ratings of the C_4 and C_5 criteria are evaluated in the following scale shown in Table 2.

Table 2: Linguistic rating based on intensity of importance

Significance	Abbreviation	Intensity of importance
Very Poor	VP	0
Poor	P	0.2
Medium Poor	MP	0.35
Fair	F	0.5
Medium Good	MG	0.65
Good	G	0.8
Very Good	VG	1

Using Table 1 for C_1, C_2 and C_3 and the corresponding discrete values of linguistic variables C_4 and C_5 (shown in Table 2) of performance rate of each alternative is shown in Table 3.

Table 3: Intensity of importance rating for each alternative correspond to attributes

Closed-Bids	Criteria				
	Price	Delay	MTBF	Alliance	Specification
	C_1	C_2	C_3	C_4	C_5
B_1	125.95	1	200000	0.5	0.35
B_2	147.37	3	250000	0.5	0.35
B_3	146.38	4	166667	0.8	0.2
B_4	129.66	2	142857	0.5	0.5
B_5	151.46	2	250000	0.35	0.5
B_6	180.25	1	250000	0.65	0.8
B_7	168.00	2	500000	0.2	0.5
B_8	125.59	4	111111	0.2	0.65
B_9	125.85	6	83333	0.5	0.35
B_{10}	176.15	2	500000	0.5	0.2

The performance ratings of the quantitative attributes C_1, C_2 and C_3 are normalized into the range of [0–1]. The normalized value of all attributes provided by 10 submitted closed-bids data given in Table 3 is set to the range of [0–1]. The normalized values are shown in Table 4.

Table 4: Normalization (N_{ij}) of decision matrix

Closed-Bids	Criteria				
	Price	Delay	MTBF	Alliance	Specification
	C_1	C_2	C_3	C_4	C_5
B_1	0.085	0.037	0.082	0.106	0.080
B_2	0.100	0.111	0.102	0.106	0.080
B_3	0.099	0.148	0.068	0.170	0.045
B_4	0.088	0.074	0.058	0.106	0.114
B_5	0.103	0.074	0.102	0.074	0.114
B_6	0.122	0.037	0.102	0.138	0.182
B_7	0.114	0.074	0.204	0.043	0.114
B_8	0.085	0.148	0.045	0.043	0.148
B_9	0.085	0.222	0.034	0.106	0.080
B_{10}	0.119	0.074	0.204	0.106	0.045

Using Eq. 1, element N_{11} (i.e., B_1C_1) is obtained: $125.95/(\sqrt{125.94^2 + 147.37^2 \ldots + 176.15^2}) = 0.085$. The entropy method utilizes the performance rate of an alternate in regards to criteria to compute the weights of various conflicting attributes without involvement of decision makers.

The main advantage of Entropy method over the conventional methods is the minimizing the subjectivity of decision maker in determining the weights and is very useful in the cases when decision makers conflicts on the values of weights alternatives [13]. After calculation (using Eq. 6-8), the information entropy of indicators are E_1, E_2, … E_n and the information entropy weights (w_j) of every criteria are shown in Table 5.

Table 5: Entropy weight of each attribute

	Price	Delay	MTBF	Alliance	Specification
	C_1	C_2	C_3	C_4	C_5
Ej	0.9960	0.9378	0.9354	0.968	0.964
Dj	0.0040	0.0622	0.0646	0.032	0.036
Entropy Weight, wj	0.0201	0.3123	0.3245	0.161	0.182
	2.01%	31.23%	32.45%	16.06%	18.25%

Table 5 (Col. 2, Row 3): E_j value is obtained using Eq. 6. $E_j = -k\sum_{j=1}^{n} N_{ij}\ \ln N_{ij}$. The constant, $k = 1/\ln(m)$ and the number of alternatives $m = 10$.

$E_1(C_1) = (0.085 \times \ln 0.085) + (0.10 * \ln 0.10) \pm \ldots + 0.119 * \ln 0.119)/\ln(10) = 0.9960$.

Table 5 (Col. 2, Row 4): Eq. 7. $D_j = (1 - E_j) = (1 - 0.9927) = 0.0073$.

Table 5 (Col. 2, Row 5): The w_j value is obtained using Eq. 8, $w_j = D_j/\sum_{j=1}^{n} D_j$,

Therefore, $w_j = D_j/\sum_{j=1}^{n} D_j = 0.0040/(0.0040 + 0.0622 + 0.0646 + 0.032) = 0.2075$.

The Bayesian method is based on the belief that importance of a criterion is a function of the decision makers or the experts' use their knowledge and cognition, and current data with the uncertain information. However, the performance of Bayesian updating can be improved by utilizing the entropy weights derived from the data and the decision makers' judgement on weights as the prior, integrate together as the Bayesian entropy weights. Four sets of weight vector evaluated by the Bayesian process is shown in Table 6.

Table 6: Prior weight vector by Experts

	Bayesian process	Price C_1	Delay C_2	MTBF C_3	Alliance C_4	Specification C_5
Set 1	*Prior 1*	10	30	30	15	15
	Posterior 1	1.30%	34.00%	42.30%	10.50%	11.90%
Set 2	*Prior 2*	10	25	30	15	20
	Posterior 2	0.80%	32.80%	40.90%	10.10%	15.30%
Set 3	*Prior 3*	20	20	20	20	20
	Posterior 3	2.00%	31.80%	32.40%	16.10%	18.20%
Set 4	*Prior 4*	30	25	20	15	10
	Posterior 4	3.20%	40.80%	33.90%	12.60%	9.50%

The next step is to obtain the weighted normalization matrix value by multiplying the normalized values obtained in Table 4 with the weight factors obtained in both entropy method and Bayesian method. The weights obtained by the entropy method and four cases of Bayesian (prior-posterior) weights provide the five sets of weighted normalization matrix. Weighted normalization matrix multiplying the Entropy weights and normalized matrix is shown in Table 7.

Table 7: Weighted normalization matrix (Entropy weights only)

Closed-Bid	Criteria				
	Price C_1	Delay C_2	MTBF C_3	Alliance C_4	Specification C_5
B_1	0.0017	0.012	0.026	0.017	0.015
B_2	0.0020	0.035	0.033	0.017	0.015
B_3	0.0020	0.046	0.022	0.027	0.008
B_4	0.0018	0.023	0.019	0.017	0.021
B_5	0.0021	0.023	0.033	0.012	0.021
B_6	0.0025	0.012	0.033	0.022	0.033
B_7	0.0023	0.023	0.066	0.007	0.021
B_8	0.0017	0.046	0.015	0.007	0.027
B_9	0.0017	0.069	0.011	0.017	0.015
B_{10}	0.0024	0.023	0.066	0.017	0.008

In the next step, the distance of each alternative from the positive ideal solution and negative ideal solution are calculated for both Entropy process and Bayesian process is shown in Table 8. In classical TOPSIS the selection is based on the principle that the chosen alternative should have the longest distance from the negative-ideal solution i.e. the solution that maximizes the cost criteria and minimizes the benefits criteria; and the shortest distance from the positive-ideal solution i.e. the solution that maximizes the benefit criteria and minimizes the cost criteria [4].

Table 8: Positive ideal solution and negative ideal solution

Bid	Entropy process		Bayesian process							
			Set 1		Set 2		Set 3		Set 4	
	S^+	S^-	S^+	S^-	S^+	S^-	S^+	S^-	S^+	S^-
B_1	0.073	0.020	0.083	0.022	0.080	0.021	0.074	0.020	0.087	0.018
B_2	0.052	0.034	0.059	0.039	0.058	0.038	0.053	0.034	0.058	0.039
B_3	0.056	0.042	0.065	0.043	0.064	0.041	0.056	0.042	0.057	0.049
B_4	0.068	0.021	0.080	0.019	0.078	0.020	0.069	0.021	0.079	0.020
B_5	0.060	0.028	0.068	0.033	0.066	0.032	0.061	0.028	0.071	0.029
B_6	0.067	0.037	0.076	0.034	0.074	0.036	0.068	0.037	0.083	0.029
B_7	0.052	0.058	0.053	0.073	0.051	0.071	0.053	0.058	0.063	0.060
B_8	0.060	0.040	0.073	0.040	0.071	0.040	0.060	0.040	0.064	0.047
B_9	0.059	0.059	0.073	0.063	0.071	0.061	0.059	0.060	0.059	0.076
B_{10}	0.054	0.057	0.053	0.073	0.053	0.071	0.054	0.057	0.062	0.060

The calculation of the S_i^+ and S_i^- follow Eq. (4a) and (4b).

$$S_i^+ = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^+\right)^2} \quad (v_{ij} \text{ is an element, } v_j^+ \text{ is max. value in a column, Table 7})$$

$$S_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2} \quad (v_{ij} \text{ is an element, } v_j^- \text{ is min. value in a column, Table 7})$$

From Table 7 (Col 2): Attribute C_1 (*Price*): $v_j^+ = 0.0025$, $v_j^- = 0.0017$

Entropy process (Col 2 & 3):

$$S_i^+ = \sqrt{(0.0017 - 0.0025)^2 + (0.0020 - 0.0025)^2 + \cdots + (0.0024 - 0.0025)^2} = 0.073$$

$$S_i^- = \sqrt{(0.0017 - 0.0017)^2 + (0.0020 - 0.0017)^2 + \cdots + (0.0024 - 0.0017)^2} = 0.020$$

The relative closeness of the alternate bids B_i with respect to S^+ and S^- has defined in Eq. 5 as $R_i = S_i^- / (S_i^+ + S_i^-)$ for $i = 1, 2, \ldots, m$.. Since $S^+ > 0$ and $S^- > 0$, the relative closeness index, $R_i \in [0, 1]$. The closeness index using the latest weight factors derived by Entropy technique and Bayesian technique is shown in Table 9.

Table 9: Closeness index (R_i) by Entropy method and Bayesian theory

Sealed-Bids	Entropy technique	Bayesian process			
		Set 1	Set 2	Set 3	Set 4
B_1	0.2105	0.2071	0.2080	0.2086	0.1736
B_2	0.3942	0.3981	0.3952	0.3944	0.4004
B_3	0.4288	0.3965	0.3905	0.4316	0.4664
B_4	0.2388	0.1943	0.2037	0.2383	0.2031
B_5	0.3197	0.3254	0.3295	0.3178	0.2872
B_6	0.3541	0.3111	0.3285	0.3508	0.2590
B_7	**0.5253**	**0.5817**	**0.5812**	**0.5214**	**0.4877**
B_8	0.3961	0.3540	0.3613	0.3989	0.4217
B_9	0.4999	0.4645	0.4617	0.5047	0.5634
B_{10}	0.5167	0.5786	0.5706	0.5133	0.4907

Using $S_i^+ = 0.073$ and $S_i^- = 0.020$ from Table 8 (Column 2, 3) under Entropy weights, the closeness index, the R_I for bid B_1 $(0 \leq R_i \leq 1)$: $S_i^- / (S_i^+ + S_i^-) = 0.073/(0.073 + 0.020) = 0.2105$.

In Bayesian weight evaluation process, decision makers or experts' past information about the significance of the criteria and attribute weight obtained from current data conveyed from the bid processed by the Entropy method is used. According to the closeness coefficient, ranking the preference order of all alternatives and methods are shown in Table 10.

Table 10: Ranking the preference order

Sealed-Bids	Entropy technique	Bayesian process			
		Set 1	Set 2	Set 3	Set 4
B_1	10	9	9	10	10
B_2	6	5	4	6	6
B_3	4	4	5	4	4
B_4	9	10	10	9	9
B_5	8	7	7	8	7
B_6	7	8	8	7	8
$\boldsymbol{B_7}$	**1**	**1**	**1**	**1**	3
B_8	5	6	6	5	5
B_9	3	3	3	3	**1**
$\boldsymbol{B_{10}}$	2	2	2	2	2

The result comprises alternate rank orders using the benchmarked attributes and data, weight factors and decision makers' view if there any changes on bidder's performance regarding any attribute in the decision-making process.

Conclusion

This study presents the hybrid framework of TOPSIS algorithm integrated with the Entropy technique and Bayesian method to address the complexity of decision making using uncertain attribute data and information. The procedure includes two weight coefficient methods: Entropy weights derived from the attribute data and Bayesian weights integrated with Entropy weights and experts' prior knowledge on weights. The priority rank orders of the alternatives are derived from the both weight coefficient methods and compared. A closed-bid supply source data with a number of tangible and intangible attributes is used to illustrate the procedure and to evaluate the best alternative. The method is decomposed into sections to understand and solve efficiently. The other bidders B7, B10 and B9 are listed at the top. The bidder B7 is ranked 1 in both Entropy weight technique and the Bayesian-Entropy integrated weights technique, except for Set 4. In Set 4, the only exception shown in B9 where decision makers' made significance change on performance rating. The results in the Bayesian-Entropy approach indicates the historical performance of a bidder on the attributes has the strong influence over the decision makers' view regarding any performance changes in determining the appropriate supply sources.

As noted above, the proposed method aims to demonstrate a procedure how to integrate any new changes on the performance (human view) into a model combined with past performance data. It provides an advantage of using an updated (posterior) information to determine potential sources upon analyzing different viewpoints and new evidences in this increasingly competitive market environment. The decision makers can have a better understanding on the impact of any new evidence, proof, change marks (such as business status change or new price quote) on the priority rank orders of alternatives. The numerical example shows that the proposed decision approach allows exploring and directing opinions and belief on the change of performance rating in a systematic approach. The methodology has the implication not only in the business sectors and manufacturing companies, it is also applicable to government sectors and non-profits that deal with multi-criteria based source selection with data uncertainty such as new policy, subcontractors and research projects.

It is thus expected to extend the methodology to the scenario in which the information of an attribute is not fully obtainable, or an event related to an attribute is incomplete, or new evidence may cause a significant change on performance rating of an alternate while comparing and selecting the best choice.

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