Performance of FSO System over Double Weibull Turbulence with Pointing Error

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Abstract

Starting with the double Weibull (DW) as turbulence-induced fading model in FSO systems, bit error rate (BER) and outage probability (OP) analysis are proposed using intensity-modulation / direct-detection (IM/DD) with binary phase shift keying (BPSK). Closed form probability density function that models the irradiance fluctuations and signal-to-noise ratio was obtained assuming quasi static turbulence channel. Novel closed-form expressions for evaluating the BER and OP performance are derived taking into account the effects of turbulence severity and pointing error. Bit error rate and OP results are obtained and displayed for moderate to strong turbulence severity and different jitter variances.

Introduction

Wireless communication technologies have experienced an extensive growth in the past few decades and the demand for technologies with low-cost, low-power, more secured communication and higher data rates was becoming substantial. Free-space optical communication have emerged as a promising wireless communication scheme which offers a license -free spectrum with very high security, low cost and low power [2], [3]. FSO creates pear-to-pear communication links through the atmosphere and provides high data rates of 1-2 Gaps over a long distance that can go up to 5 Km. However, this comes at the expense of major degradation in the performance of the FSO communication systems that might happen due to atmospheric turbulence including temperature and pressure changes [4]. Severe challenges can arise from the effect of these atmospheric disturbances and accordingly, the atmospheric channels are modeled with statistical distributions that describe their characteristics, such as Gamma-Gamma [5], double Weibull [1] and so on. In this paper, we use the double Weibull distribution to analyze the FSO communication systems under moderate to strong turbulence conditions. It has been proven by [1], for moderate to strong turbulence conditions the double Weibull distribution is more reliable and more accurate than the Gamma-Gamma distribution. Furthermore, we study the impact of the misalignment (pointing error) between the transmitter and the receiver due to building sway [6]-[9]. In this paper, we propose a new FSO system model that accounts for the combined effect of Double Weibull atmospheric turbulence and of pointing error.

In this work, we use the analytical framework proposed by Gappmair [6] to show the effect of pointing errors on the distribution of the irradiance proposed by Chatzidiamantis et al. [1]. Moreover, we derive a new closed-form expressions to describe the channel statistics as well as the random behavior of the received signal-to-noise ratio (SNR). The derived expressions are obtained for a single FSO link influenced by slowly fading turbulence channel and pointing error. The optical system assumes a plane wave with intensity modulation/direct detection technique (IM//DD) with binary shift-keying (BPSK). Based on the derived CDF and PDF of the received NSR, performance metrics evaluation such as average bit error rate, and outage probability (OP) of the proposed FSO system over the double Weibull fading channels is obtained.

The remaining of the paper is organized as follows; in next section, we propose the PDF of the irradiance fluctuations under the impact of pointing errors over double Weibull turbulence channel model. In the following two sections, we derive the statistical model of a single FSO link signal to noise ratio followed by closed expressions for the bit error rate and the outage probability. Next, we provide some numerical results and conclude the paper.

Channel and System Models

This paper studies a single FSO link with intensity modulation/direct detection (IM/DD) detection technique. The transmission of the data can be affected by many factors, such as; path loss, atmospheric turbulence conditions, pointing errors, and additive white Gaussian noise (AWGN) that can be modeled as

$$y = \eta I x + w \tag{1}$$

where η is the actual photoelectric conversion ratio, x is the transmitted intensity and w is an AWGN sample with one-sided power spectral density N_0 W/Hz. Moreover, I is the channel gain that can be modeled as $I = I_0 I_a I_p$, in which I_0 is the path loss weight (i.e. depends on distance and weather conditions and normalized to 1), I_p is the pointing error effect and I_a is the turbulence-induced fading[10].

Farid and Hranilovic [10], derived a statistical model for an FSO system influenced by gamma-gamma atmospheric turbulence and pointing error. Their model took into account the effect of turbulence channel severity as well as the beam width, detector size, and pointing error variance. In our paper, in modeling the effect of pointing error on the system performance, we will adopt the model used by [10], which suggest the random distribution I_p due to pointing error can be modeled as Rayleigh distribution and is given by [10, Equ. (11)]

$$f_{I_p}(I_p) = \frac{\xi^2}{A_0^{\xi^2}} I_p^{\xi^2 - 1}, 0 \le I_p \le A_0 \qquad (2)$$

where A_0 is a constant that outlines the pointing loss. The parameter $\xi = \frac{w_e}{2\sigma_s^2}$, is the ratio between the equivalent laser beam radius at the receiver w_e, and the pointing error jitter (i.e. displacement standard deviation) at the receiver. Fig. (1) [3], shows how the pointing error jitter and the equivalent beamwidth affect the parameterξ. It is obvious from the fi

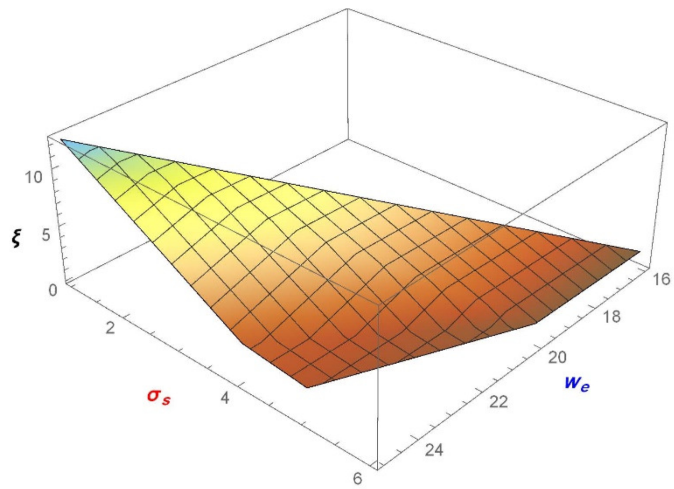

Figure 1: Pointing error severity factor ξ Vs. beamwidth and jitter standard deviation.

Chatzidiamantis *et. al.* [1, Equ. (5)] presented a closed form expression for the double Weibull irradiance distribution in terms of Meijer's G function, without accounting the effect of pointing error. In their analysis, the double-weibull random variable is modeled as the product of two weibull distributed random variables each with different β, Ω. The double Weibull random variable has the form $I_a = I_x I_y$, such that $I_x \sim Weibull(\beta_1, \Omega_1)$ *and* $I_y \sim Weibull(\beta_2, \Omega_2)$ with PDF taken as the form [1, Equ. (5)]

$$f_{I_a}(I_a) = \frac{\beta_2 K\, L^{0.5} K^{0.5} (2\pi)^{1 - \frac{(K+L)}{2}}}{I_a} \cdot G_{K+L,0}^{0,+K+L}\left\{ K^K\, L^L\, \Omega_1^L \left(\frac{\Omega_2}{I_a^{\beta_2}}\right)^K \middle|\, \begin{matrix} 1 - k_0 \\ - \end{matrix} \right\} \qquad (3)$$

where $k_0 = \Delta(L : 0), \Delta(K : 0), G_{p,q}^{m,n}$ [.] is the Meijer G function, K and L are positive integers such that $\frac{L}{K} = \frac{\beta_2}{\beta_1}$ *and* $\Delta(x : y) \triangleq \frac{y}{x}, \frac{y+1}{x} \cdots, \frac{y+x-1}{x}$. Therefore, the Probability joint distribution of $I = I_a I_p$ is given by [6], [7].

$$f_I(I) = \int_{I_p} f_{I|I_p}(I|I_p) f_{I_p}(I_p) dI_p \qquad (4)$$

With the conditional probability $f(I|I_p)$ is given by

$$f(I|I_p) = \frac{1}{I_p} f I_{\mathrm{a}}\left(\frac{I}{I_p}\right), \qquad 0 \le I_p \le A_0 \tag{5}$$

Inserting (2) and (3) in (4), and by using [11, Equ. (07.34.21.0084.01)], a new expression for the received irradiance pdf taking into account the combined effects of turbulence and pointing error is found having the form

$$f_I(I) = \frac{\xi^2 K^{0.5}\ L^{0.5}\ (2\pi)^{1-\frac{(K+L)}{2}}}{I} \times G^{\,0,\beta_2 K+K+L}_{\,\beta_2 K+K+L,\beta_2 K}\left\{ K^K\, L^L\, \Omega_1^L\, \Omega_2^K \left(\frac{A_0}{I}\right)^{\beta_2 K} \middle|\, \begin{matrix} k_1 \\ k_2 \end{matrix} \right\} \tag{6}$$

Where $k_1 = \Delta(\beta_2 K : 1 - \xi^2), \Delta(L : 0), \Delta(K : 0)$ *and* $k_2 = \Delta(\beta_2 K : -\xi^2)$. The parameter β_2 must be an integer or half integer such that $\beta_2 K$ remains positive integer. It is important to note that when pointing error effect approaches zero (i.e. $\xi \to \infty$ and $A_0 \to 1$), eq. (6) coincides mathematically and numerically with [1, Equ. (5)].

Electrical Signal to Noise Ratio

This section includes the statistics of the electrical SNR over double Weibull channel bearing in mind the pointing error effect for IM/DD with BPSK. Under this type of detection, the average electrical SNR μ is given by $\mu = \frac{(\eta \mathrm{E}[I])^2}{N_0}$[6]. , where $\mathrm{E}[I]$ is the average value of the received irradiance with respect to (6), by using [11,Equ. (07.34.21.0088.01)], an expression for the average electrical SNR is given by

$$\mathrm{E}[I] = \frac{A_1 B_1 A_o}{(1+\xi^2) A_2^{\frac{1}{\beta_2 K}}} = h A_o \tag{7}$$

The notation $\mathrm{E}[\cdot]$ symbolizes the expectation with respect to Equation (6), where $A_1 = \xi^2 K^{0.5}\ L^{0.5}\ (2\pi)^{1-\frac{(K+L)}{2}}$, $A_2 = \frac{1}{K^K\, L^L\, \Omega_1^L\, \Omega_2^K}$, and $B_1 = \prod_{i=1}^{L+K} \mathrm{Gamma}(\frac{1}{\beta_2 K} + k_{2,i})$. The parameter $h = A_1 B_1 / (1+\xi^2) A_2^{\frac{1}{\beta_2 K}}$. By realizing the instantaneous SNR Υ defined as $\Upsilon = \frac{(\eta I)^2}{N_0}$ [12], and $\frac{\eta^2}{N_0} = \frac{\mu}{h^2 A_0^2}$, the irradiance can be written as: $I^2 = \frac{\Upsilon h^2 A_0^2}{\mu}$, by using the value of I^2 and applying the transformation method on (6), an expression for the instantaneous SNR PDF is found and has the form:

$$f_{\Upsilon}(\Upsilon) = \frac{\xi^2 K^{0.5}\ L^{0.5}\ (2\pi)^{1-\frac{(K+L)}{2}}}{2\Upsilon} \cdot G^{\,0,\beta_2 K+K+L}_{\,\beta_2 K+K+L,\beta_2 K}\left\{ \frac{K^K\, L^L\, \Omega_1^L\, \Omega_2^K}{h^{\beta_2 K}} \left(\frac{\mu}{\Upsilon}\right)^{\frac{\beta_2 K}{2}} \middle|\, \begin{matrix} k_1 \\ k_2 \end{matrix} \right\} \tag{8}$$

The cumulative distribution of SNR $F_{\Upsilon}(\Upsilon)$, can be found from (8) as $F_{\Upsilon}(\Upsilon) = \int_0^{\Upsilon} f_{\Upsilon}(\Upsilon) d_{\Upsilon}$. By using [11, Equ. (07.34.21.0084.01)] an expression for the SNR cumulative distribution is found and has the form

$$F_{\Upsilon}(\Upsilon) = \frac{\xi^2 K^{0.5} \; L^{0.5} \; (2\pi)^{1-(K+L)}}{\beta_2 K} \times G^{\,2(\beta_2 K+K+L),\beta_2 K}_{\,3\beta_2 K,2(\beta_2 K+K+L)+\beta_2 K}\left\{ C\left(\frac{\Upsilon}{\mu}\right)^{\beta_2 K} \middle| \begin{matrix} k_3 \\ k_4 \end{matrix} \right\} \quad (9)$$

Where $C = \left(\frac{A_2 h^{\beta_2 K}}{2^{K+L}}\right)^2$, $k_3 = \Delta(\beta_2 K : 0), \Delta(2 : 1 - \frac{-\xi^2}{\beta_2 K}), \ldots, \Delta(2 : 1 - \frac{\beta_2 K - 1 - \xi^2}{\beta_2 K})$, where k_3 comprising $3\beta_2 K$ terms. The parameter $k_4 = \Delta\left(2 : 1 - \frac{1-\xi^2}{\beta_2 K}\right), \ldots, \Delta\left(2 : 1 -\right.$ $\beta 2_{K-\xi 2} \beta 2_{K, \Delta 2:1, \ldots, \Delta 2:1-L-1L, \Delta 2:1, \ldots, \Delta 2:1-K-1K}$, in which $k4$ comprising $2K + L + \beta 2_{K} + \beta 2_{K}$ terms.

Average Bit Error Rate

Based on the formula of the average BER [13, Equ. 12]

$$\bar{P}_{\mathrm{b}} = \frac{q^p}{2\Gamma(p)} \int_0^{\infty} \exp\left(-q\,\Upsilon\right) \Upsilon^{p-1} F_{\Upsilon}(\Upsilon)\, d\Upsilon \quad (10)$$

Where $\Gamma(*)$ is the well known gamma function. The parameters p and q determine modulation scheme type and their values for different modulation schemes can be founded in [13-14]. By placing (9) into (10) and utilizing [11, Eq. (07.34.21.0088.01), the average BER $\bar{\mathrm{P}}_{\mathrm{b}}$ has the form

$$\bar{P}\mathrm{b} = \frac{\xi^2 K^{0.5} \; L^{0.5} \; (2\pi)^{0.5(3-\beta_2 K-2(K+L))}}{2\Gamma(p)\; (\beta_2 K)^{1.5-p}}$$

$$\times \; G^{\,2(\beta_2 K+K+L),2\beta_2 K}_{\,3\beta_2 K,2(\beta_2 K+K+L)+\beta_2 K}\left\{ C\left(\frac{\beta_2 K}{q\,\mu}\right)^{\beta_2 K} \middle| \begin{matrix} \Delta(\beta_2 K, 1-p) \\ k4 \end{matrix} \right\} \quad (11)$$

Outage Probability

In fading channels as the case of FSO system under study, outage is encountered when the received instantaneous SNR (Υ) falls below a minimum acceptable value (Υ_{th}). When the outage occurs, the system performance becomes unreliable and unacceptable in which the received symbols cannot be recognized with a small probability of error [15]. A system metric termed probability of outage and is evaluated using

$$P_{out} = P_r(\Upsilon < \Upsilon_{th}) = F_{\Upsilon}(\Upsilon_{th}) \quad (12)$$

Numerical Example

In this section, we prove the validity of our analytical results by presenting some selected numerical examples. The impact of pointing errors, turbulence conditions effect are taken into account on the performance of a single FSO link deploying BPSK with IM/DD detection systems for plane waves data.

Table 1, shows the system and channel parameters used in this section to the generate the numerical results. The ratio L_o/R_F given by table 1, represents the size of the inner scale scintillation with respect to the first Fresnel zone. The rest of the parameters are related to the double-weibull pdf parameters and can be evaluated using [1, Equations 8-9]. In table 1, it is assumed that β_2 is an integer to satisfy the condition $\beta_2 K$ being an integer.

Figures 2 and 3, depicts BER and outage probability results respectively for different values of pointing error for moderate to strong turbulence conditions. Furthermore, the results are presented as a function of received electrical average signal to noise ratio μ. The figures show the impact of turbulence severity and pointing error on the BER and outage probability performance. As moving from moderate to strong turbulence conditions, the BER and the outage probability values increases, leading to poorer performance on both system metrics. The same conclusion can be drawn for the case when the pointing error value increases.

Table 1: System and channel parameters for plane wave data [1]

	σ_R^2	L_o/R_F	β_1	β_2	l/k	Ω_1	Ω_2
Moderate	2	0.5	1.522	1	6/9	1.171	1.114
Strong	25	1	1.252	1	4/5	1.093	1.006

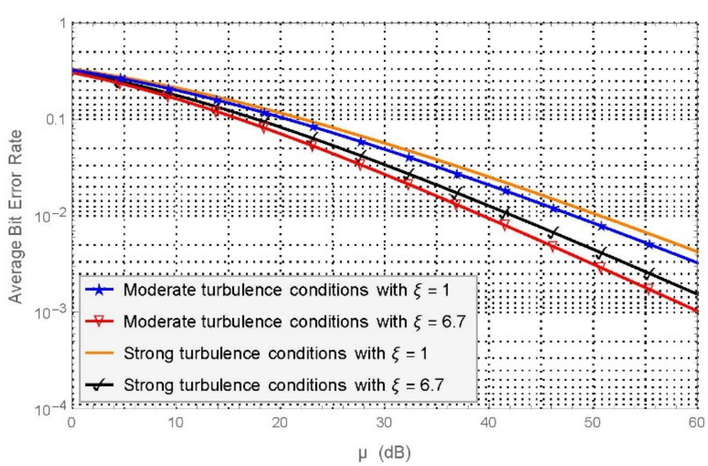

Figure 2: The impact of pointing errors on the average bit error rate of a single

FSO link under strong and moderate turbulence with $p = q = 1.0$

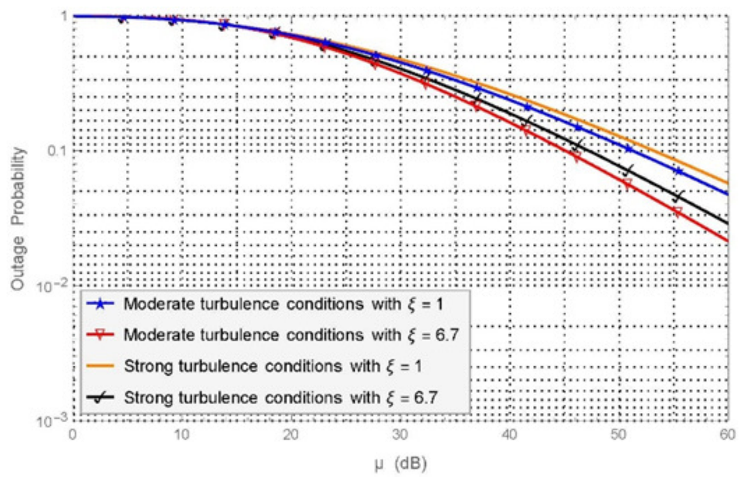

Figure 3: The effect of pointing errors on the outage probability of a single FSO link under strong and moderate turbulence with $\Upsilon_{th} = 20dB$ *,* $\xi = 1, 6.7$*, and p =1 and q =1.*

In general, turbulence conditions cause a degradation in the error performance. Fig. [2-3] shows the impact of pointing errors on the performance of the link. Both Figures shows that as $\xi \to \infty$ (the effect of pointing error approaches zero), the performance becomes better.

Conclusion

Closed form expressions for two FSO system metrics over Double-Weibull turbulence channel with pointing error were derived in terms of the Meijer's G function. The derived expressions were developed for outage probability and bit-error rate. In addition, the system metrics were developed for intensity modulation with direct detection.

The derived expressions where then used to evaluate the system performance for the two system metrics for IM/DD detection scenario. The results were displayed for different turbulence strength and pointing error severity as a function of the received average signal-to-noise ratios. By inspecting these results, an increase in turbulence strength and/or the pointing error tends to degrade the system performance for both system metrics.

It is of a great interest to study the effect of fast fading turbulence channels on the present proposed system. Error correcting codes are known for its ability to improve system metrics performance, so, we suggest for future studies to evaluate the performance of the present system using iterative decoding. In addition, it will be also a great interest to test how will different diversity techniques can improve the performance of the present system.

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