Outage Results for Heterodyne Differentially Encoded PSK over FSO Turbulence Channel with Pointing Errors

Omar M. Hasan
Princess Suamay University for Technology
Amman. Jordan
ohasan@psut.edu.joed

Akram Abu Aisheh
University of Hartford,
West Hartford, Connecticut
abuaisheh@hartford.edu

Abstract

Outage probability closed form solution is derived for heterodyne differentially encoded PSK (DEPSK) over free-space atmospheric channel. The channel is assumed to be gamma-gamma distributed, slowly fading with pointing error due to building sway. Furthermore, the channel is assumed to be quasi-static, therefore, the effect of phase noise between two consecutive bit intervals is ignored in the analysis. The derived mathematical expression allows the evaluation of the results as a function of channel strength, weak to strong, beam width and jitter variance. Current results show that, for smaller jitter variances less than ≈ 0.8, better outage results can be attained for beamwidths smaller than the jitter variance, while for jitter variances greater than ≈ 1.0, better outage performance can be attained for beamwidth values larger than the jitter variances.

Introduction

Recently, Free Space Optical communications (FSO) has received considerable attention due to its attractive characteristics and its ability to meet growing demands for high speed data rate applications such as 4G/LTE, cloud computing and video transmission. FSO needs no licensing for deployment and can be easily deployed with speed of 1-2 Gb/sec over link distances in the range of 1km - 2km. However, FSO atmospheric channel can highly degrade FSO system performance due to atmospheric turbulence [1-4], causing signal fading (scintillation), pointing error [5-6] and weather conditions [7]. Turbulence [4] is caused by refractive index variations along the propagation path and its strength can vary from weak to strong depending on the Rytov variance σ_R^2 value. In weak turbulence conditions, the Rytove variance $\sigma_R^2 \ll 1$, while for $\sigma_R^2 > 1$, turbulence strength varies from medium to strong turbulence regimes until it enters the saturation region $\sigma_R^2 = 25.0$.

In working with FSO systems, many statistical channel models can be used to describe the effect of turbulence channel. However, the appropriate usage of the channel model depends

ISBN 978-1-60643-379-9

on the turbulence strength [8-11]. It was shown in Figures 1-4 [12], that the gamma-gamma distribution model is valid for all turbulence strength conditions, weak to strong regimes.

In this paper, we study outage probability performance for DEPSK-FSO system operating over Gamma-Gamma atmospheric channel influenced by the combined effect of turbulence and pointing error. To do so, a novel closed form expression for the system outage probability is derived taking into account the combined effect of turbulence conditions and pointing error.

The paper is organized as follows. In the first section, we study and analyze the channel under study and derive a closed form expression for the system outage probability. Following this section, we provide numerical results for differentially encoded phase-shift keying (DEPSK) system outage probability for different turbulence severity, different beamwidths and jitter variances. The final section of the paper, summarizes our work and provides our comments and conclusions.

System and Channel Model

In this paper we consider an FSO system using heterodyne DEPSK modulation operating over Gamma-Gamma turbulence channel. The DEPSK signal is assumed to propagate along a horizontal path over slowly fading turbulence channel and the received signal suffers from pointing errors due to building sway. The received electrical signal r will have the following form

$$r = hx + n \tag{1}$$

Where x is the transmitted bits, n is additive white gaussian noise process with one sided power spectral density N_0 W/Hz and variance $\sigma_n^2 = N_0/2$ W/Hz. The parameter $\boldsymbol{h}$ is the channel state and assumed to be the product of two independent factors:

$$h = h_a h_p \tag{2}$$

Where h_a is the random attenuation due to the atmospheric turbulence modeled by the gamma-gamma probability density function as

$$f_{h_a}(\boldsymbol{h}) = \frac{2\cdot\alpha\beta(\boldsymbol{\alpha\beta h_a})^{\frac{\alpha+\beta}{2}-1}}{\Gamma(\alpha)\Gamma(\beta)} \; K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\,\right), h_a > 0 \tag{3}$$

The parameters α and β represent the effective number of large-scale and small scale turbulence cells given by

$$\alpha = \frac{1}{\sigma_x^2}\,, \beta = \frac{1}{\sigma_y^2} \tag{4}$$

The quantities σ_x^2 and σ_y^2 are the large scale and small scale scattering variances. For plane wave propagation mode, large and small scale variances in the case of zero inner scale are found using [12, Eqs. 18-19]

$$\sigma_x^2 = exp\left[\frac{0.49 \cdot \sigma_R^2}{(1+1.11 \cdot \sigma_R^{12/5})^{7/6}}\right] - 1 \quad (5)$$

$$\sigma_y^2 = exp\left[\frac{0.51 \cdot \sigma_R^2}{(1+0.69 \cdot \sigma_R^{12/5})^{5/6}}\right] - 1 \quad (6)$$

where σ_R^2 is the Rytov variance given by

$$\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6} \quad (7)$$

Here, $k = 2\pi/\lambda$ is the optical number, λ is the wavelength, and C_n^2 represents the index of refraction structure parameter while L is the link propagation path.

The second term h_p in eq. (2) is a random attenuation due to pointing error given by [4]

$$f_{h_p}(h_p) = \frac{\gamma^2}{A_o^{\gamma^2}} h_p^{\gamma^2 - 1} \quad (8)$$

where $\gamma = w_{zeq}/2\sigma_s$, is the ratio between the equivalent beam radius at the receiver and the jitter standard deviation, where $w_{zeq}^2 = w_z^2 \sqrt{\pi}\,\mathrm{erf}(v)\,/2v exp(-v^2)$, $v = \sqrt{\frac{\pi r^2}{2 w_z^2}}$, and $A_o = [\mathrm{erf}(v)]^2$. It was shown by [6], the probability density function of the random process given by eq.(2) can be written as

$$f_h(h) = \frac{\alpha\beta\gamma^2}{A_o \Gamma(\alpha)\Gamma(\beta)} \cdot G_{1,3}^{3,0}\left[\frac{\alpha\beta}{A_o} \middle| \begin{matrix} \gamma^2 \\ \gamma^2 - 1, \alpha - 1, \beta - 1 \end{matrix}\right] \quad (9)$$

Where G is the Meijer's G-function as in [14-15].

Outage Probability Derivation

Outage occurs when the received SNR being u falls below the threshold SNR value allowed by the system, u_{min}. When outage takes place, the outage probability can be expressed as

$$P_{out} = P_r(u < u_{min}) = F_u(u_{min}) \quad (10)$$

By using [14, eq.(26)], an expression for the outage probability is found having the form

$$P_{out} = \frac{\gamma^2}{\Gamma(\alpha)\Gamma(\beta)} G_{2,4}^{3,1}\left[\frac{\alpha\beta\gamma^2}{1+\gamma^2} \frac{u_{min}}{\bar{u}} \middle| \begin{matrix} 1, 1+\gamma^2 \\ \gamma^2, \alpha, \beta, 0 \end{matrix} \right] \tag{11}$$

It is to be noted here, the effect of phase noise is ignored in the above analysis since the channel was assumed to be quasi static. Therefore, it is justified to ignore the noise phase accumulation between two consecutive bit intervals.

Numerical Results

Using eq. (11) above, we present numerical results for outage probability performance for the FSO system explained above. Fig. 1, depicts outage probability results for turbulence strengths weak to strong vs. normalized average SNR with normalized beamwidth $w_z/r = 7.0$ and jitter variance $\sigma_s/r = 3.0$. As it was expected, by keeping the beamwidh and the jitter variance fixed, better outage probability performance is attained by moving from strong to weak turbulence conditions. Fig. 2, demonstrates outage probability results for moderate turbulence condition and normalized jitter variance $\sigma_s/r = 0.80$ for normalized beamwidths $w_z/r = 2.0 - 6.0$. It is interesting to note from the figure, for $SNR < 22.5\ dB$, best performance can be achieved for $w_z/r = 2.0$, while for $SNR > 22.5\ dB$, best performance can be achieved for $w_z/r = 3.0$. Finally, Fig. 3 depicts outage probability results for weak and moderate turbulence conditions. The results are displayed for normalized beamwidth $w_z/r = 6.0$, and $\sigma_s/r = 0.80, 1.2, 1.6, 2.0$. it can be seen from the figure, better performance can be attained for smaller values of jitter variances regardless of the turbulence strength.