Modelling Fluid Flow Through a Flexible Tube

AbdessamadMEHDARI
Mohammed V University
abdmehdari@gmail.com

Mohamed HASNAOUI
Royal Air Force School
hasnaouimohammed@hotmail.com

Mohamed AGOUZOUL
Mohammed V University
agouzoul@emi.ac.ma

Abstract

We are interested, in this paper, in the modeling of the behaviour of an instationnary viscous flow in a tube with a flexible wall. This phenomenon is an interesting problem,often, encountered in many industrial systems: biological, renewable energies and recently, in the field of transporting gaseous materials under pressure. The document will provide a review of recent modeling which goals to understanding the effects of the tube wall characteristics on the fluid flow. Firstly, the Newtonian incompressible fluid will be analysed following the process of an asymptotic approach according to a large Reynolds number and a small aspect radio. Secondly, the wall has been assumed a thin shell and generates a small axisymmetric vibration. The mathematical model for the said wall is developed by using a thin shell theory. In this technique, the quadratic approach is applied to model the tube. Finally, the different parameters of the fluid and shell characteristics were studied on amplitude ratios

Introduction

The importance of studying the flow in deformable tubes is due to its occurrence within a various uses in many industrial systems. This standing is reflected in biology [1], in micro-fluidic devices [2, 3], in the renewable energies [4], and recently, in the field of transporting gaseous materials under pressure [5].

Besides the large difficulty to analytically solve the Navier-Stokes equations, our system is characterized by three types of parameters: parameters related to the rheological behavior of the fluid [6,7], parameters characterized by the nature and geometry of the wall [8,9] and hydrodynamic conditions [10].We are interested, in this paper, in the symmetric and three-dimensional flow through an elastic tube with a variable radius, in the presence of gravity force.

From another point of view, since the unicity of the solutions of the Navier-Stokes equations holds only for two-dimensional problems, it is of importance, in the three-dimensional case, to use asymptotic modelling to determine approximate solutions.

In this model, there exists a small parameter ‘ε’ characterizing the aspect ratio of the tube. This parameter governs the asymptotic expansion of the analysis.
The solution depends on the pressure behavior. An expansion of the solution is constructed by means of asymptotic tools, including the effects of several fundamental physical parameters. Some examples are considered for which we estimate the application ranges of this model. The results are matched with other results obtained by means of numerical methods and experimental results.

Formulation of the Problem

The phenomenon which is considering here is caused by the interaction of the fluid with its containe, the tube.The tube is assumed to be straight, long, with circular cross sections of variable radius. We will assume that the fluid is incompressible and Newtonian. To express the problem we will use the cylindrical coordinates r' , θ' , z' (the physical variables are denoted using primes).

In the reference configuration the length of the domain is denoted by L. We will choose the z-axis along the axis of the tube (see Figure.1.).

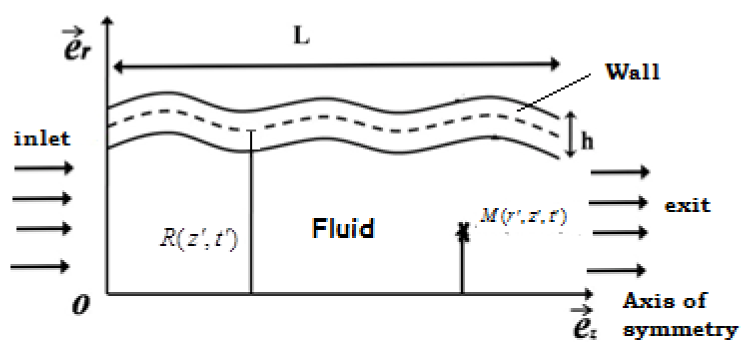


Figure 1: The deformed domain

The variable radius $R(z',t')$,is a function of the longitudinal variable and time.The lateral boundary is assumed to be elastic and to deform as a result of the interaction between the fluid and the structure. Moreover, we assume that it behaves as a homogenous, isotropic, lineary elastic membrane shell with thickness h.

Fluide

The flow is instationary and axially symmetric, in the presence of gravity force. The fluid, moving in a elastic tube, is incompressible and viscous. The motion is governed by the Navier-Stokes and the continuity equations.

At this level, we introduce scaled variables, namely:

$$\begin{cases} r=\dfrac{r'}{R_0} \quad z=\dfrac{z'}{L} \quad t=\dfrac{t'}{T_{ref}} \quad \varepsilon=\dfrac{R_0}{L} \\ U=\dfrac{U_r}{\varepsilon W_0} \quad W=\dfrac{U_z}{W_0} \quad P=\dfrac{P'}{\rho_f^0 \varepsilon^2 W_0^2} \end{cases} \qquad (01)$$

For the scaling quantities are referenced by: T_{ref}, the reference time; $\varepsilon=\dfrac{R_0}{L}$: the aspect ratio and W_0 is axial velocity. ; ρ_f^0 and υ are respectively the density and kinematic viscosity of fluid. Here U_r and U_z denote the components of the fluid velocity along r' and z' directions respectively; P' is the pressure.

Using scaled variables, the non-dimensional equations of the problem, read as:

$$\begin{cases} \left(\dfrac{S_t}{\varepsilon}\right)\dfrac{\partial U}{\partial t}+U\dfrac{\partial U}{\partial r}+W\dfrac{\partial U}{\partial z}=-\dfrac{\partial P}{\partial r}+\dfrac{\Re_e^{-1}}{\varepsilon}\left[\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial U}{\partial r}\right)+\varepsilon^2\dfrac{\partial^2 U}{\partial z^2}-\dfrac{U}{r^2}\right] \\ \left(\dfrac{S_t}{\varepsilon}\right)\dfrac{\partial W}{\partial t}+U\dfrac{\partial W}{\partial r}+W\dfrac{\partial W}{\partial z}=-\varepsilon^2\dfrac{\partial P}{\partial z}+\dfrac{\Re_e^{-1}}{\varepsilon}\left[\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial W}{\partial r}\right)+\varepsilon^2\dfrac{\partial^2 W}{\partial z^2}\right] \\ \dfrac{\partial U}{\partial \overline{r}}+\dfrac{U}{r}+\dfrac{\partial W}{\partial z}=0 \end{cases} \qquad (02)$$

The numbers figuring in (02) are the Reynolds number $\Re_e=\dfrac{R_0 W_0}{\upsilon}$ and the Strouhal number

$$S_t=\frac{R_0}{W_0 T_{ref}}.$$

Atlarge Reynolds number and low Strouhal number, the system of this study (02) is valid under the asymptotic restriction:

$$\Re_e^{-1}\equiv\varepsilon\equiv S_t \qquad (03)$$

It is relating the Reynolds, the Strouhal numbers, and the aspect ratio of the tube. The relation (03) seems to be restrictive. But, from a mathematical point of view, it characterizes a phenomenon where the initial and boundary layers originate competitive influences, and put in evidence an oscillatory phenomenon of fluid-structure system at low frequency.

Let us linearize equations (02) about the particular solution at the inlet of tube. We look U, W and P in the form:

$$U = \delta_1(\varepsilon)\tilde{U}_1, \ W = 1 + \delta_2(\varepsilon)\tilde{W}_1 \quad , \quad P = \tilde{P}_{amb} + \delta_3(\varepsilon)\tilde{P}_1 \qquad (04)$$

Where $\tilde{U}_1, \tilde{W}_1$ and $\tilde{P}_1$ are, respectively, the perturbed radial and axial velocities, and pressure:

$$\begin{cases} \tilde{U}_1 = 0 \\ \tilde{W}_1 = 0 \\ \tilde{P}_1 = 0 \end{cases} \qquad \text{for} \qquad t = 0 \quad r = 0 \quad z = 0 \qquad (05)$$

$\tilde{P}_{amb}$ is the adimensional ambient pressure in the tube before perturbation. $\delta_i(\varepsilon)$ (*i=1,2,3*) are the 'gauge functions' and determinated according the *"Least Degeneration Principle"*(It consists to keep the maximum terms in equation(02)). The gauge function is defined as:

$$\delta_i(\varepsilon) = \varepsilon^{\alpha_i} \quad \text{and} \qquad Lim_{\varepsilon \to 0}\delta_i(\varepsilon) = 0 \qquad (06)$$

Which 'α_i' is a positive real number. Here, we found: $\alpha_i = 3$.

Inserting (06) into (02), we obtain the non-degenerate adimensional equations, namely:

The 0th order ε^2 terms

$$\begin{cases} \dfrac{\partial \tilde{U}_1}{\partial t} + \dfrac{\partial \tilde{U}_1}{\partial z} = -\dfrac{\partial \tilde{P}_1}{\partial r} + \left[\dfrac{\partial^2 \tilde{U}_1}{\partial r^2} + \dfrac{\partial}{\partial r}\left(\dfrac{\tilde{U}_1}{r} \right) \right] \\ \dfrac{\partial \tilde{W}_1}{\partial t} + \dfrac{\partial \tilde{W}_1}{\partial z} = \left[\dfrac{\partial^2 \tilde{W}_1}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \tilde{W}_1}{\partial r} \right] \\ \dfrac{\partial \tilde{U}_1}{\partial r} + \dfrac{\tilde{U}_1}{r} + \dfrac{\partial \tilde{W}_1}{\partial z} = 0 \end{cases} \qquad (07)$$

The 1st order ε^2 terms

$$\begin{cases} \dfrac{\partial^2 \tilde{U}_1}{\partial z^2} = 0 \\ \dfrac{\partial^2 \tilde{W}_1}{\partial z^2} - \dfrac{\partial \tilde{P}_1}{\partial z} = 0 \end{cases} \qquad (08)$$

The appropriate boundary conditions are:
(1). Regularity of solution along z-axis, as:

$$\frac{\partial \tilde{W}_1}{\partial r} = 0, \quad \tilde{U}_1 = 0 \qquad \text{on } r = 0 \tag{09}$$

(2). Kinematic condition on the contact boundary of the fluid and the structure (will formulated later).
In the sequel, taking into account boundary conditions and initial conditions, an expansion of the solution is constructed by means of analytical tools, including the effects of several fundamental physical parameters.

Tube

The study of the rheological behavior of the tube requires the study of the complex relationship between the stress and strain tensors. For to explain said relationship, we will adopt the shell thin theory. In fact,Love (1888) [11] adopted kirchoff's assumptions for thin plate theory [12] and added to them the thin shell approximation [13]. In 1970, Koiter [14] confirmed the first coherent approximation of thin shell theory presented by Love. Nevertheless, HsuJ.C. CliftonR.J and 1956 [15] and AnlikM Izaman and 1966 [16] were the ones who exploited Love'sassumptions to study analytically the flow in an elastic pipe. These studies analyze the behavior of blood flow in the arteries. Later, in 1978 and 1986, Moddie & Haddow [17] and Bahrar [10] respectively adopted the assumptions of that theory.

The said theory is mostly presented in three theories kind named: quadratic theory, linear theory and the theory of membranes [18].

An elastic shell's behaviour under the influence of external loads (forces, moments, temperature gradients, etc.) is of course governed by the theory of Elasticity. Unfortunately, finding a solution to such a problem has been characterized by a difficult and complexe systems.

The most common shell theories are those based on linear elasticity concepts. Linear shell theories predict adequately stresses and deformations for shells exhibiting small elastic deformations; i.e., deformations for which it is assumed that the equilibrium equation conditions for deformed shell surfaces are the same as if they were not deformed, and Hooke's law applies.

Shells of revolution, a very important class of thin shells, have many technical applications in engineering.A cylinder is generated by moving a straight line along a curve while maintaining itparallel to its original position. It follows from this definition that through everypoint of the cylinder one may pass a straight line that lies entirely on this surface.These lines are called the generators. All planes that are normal to the generators intersect the cylinder in identical curves, which are called profiles. According to the Kirchhoff assumptions, deformations throughout thewhole volume of the shell material are completely defined by deformations andchanges in curvature of the middle surface. Thus, the adoption of these hypothesesreduces the three-dimensional (3D) shell problem to the two-dimensional (2D) problemof equilibrium and straining of the middle surface of a shell. So, the shell will beregarded as a 2D body, i.e., a collection of material points situated on the middlesurface.

The flexible tube of length L, undeformed radius R_0 and wall thickness h is modelled as a cylindrical shell and its deformatiok2n is described using the geometrically non-linear Kirchhoff–Love.The Kirchhoff–Love assumption states that material lines that were normal to the undeformedmidplane remain normal to the shell's midplane throughout its deformation and that they remain unstretched.

The deformation of the shell is expressed in terms of the midplane displacements. Here, we consider only the radial displacement :

$$\vec{A} = \left[R(z',t') - R_0 \right] \vec{e}_r \tag{10}$$

At this level, we introduce scaled variables, namely:

$$\begin{cases} \overline{R} = \dfrac{R}{R_0}, \overline{t} = \dfrac{t'}{T_{0.T}}, \varepsilon_0 = \dfrac{h}{R_0} \prec\prec 1, \gamma_f^0 = \dfrac{\rho_f^0}{\left(\lambda_1 + \lambda_2\right)}.W_0^2 \\ \overline{P}^* = \dfrac{P_{\text{int}-tube} - P_{ext-tube}\big|_{r'=R(z',t')-\frac{h}{2}}}{\rho_f^0.\varepsilon^2 W_0^2}, \gamma_T^0 = \dfrac{\rho_T^0}{\left(\lambda_2\right)}.\left(\dfrac{R_0}{T_{0.T}}\right)^2 \end{cases} \tag{11}$$

Where λ_1 et λ_2 are the Lame constants : $\lambda_1 = \dfrac{E}{2\left(1+\upsilon_T\right)}$ et $\lambda_2 = \dfrac{E\,\upsilon_T}{\left(1-\upsilon_T\right)}$ (E: Young's modulus and υ_T Poisson's ratio), ρ_T^0 and ρ_f^0 are respectively the density of the tube and fluid. $T_{0.T}$ is characteristic time of tube.

Thus, the non-diemsional equation for radial displacement of the tube wall is, namely:

$$\gamma_T^0 \frac{\partial^2 \overline{R}}{\partial t^2} - \frac{1}{\overline{R}}\left(1 - \frac{1}{\overline{R}}\right) - \gamma_f^0 \frac{\varepsilon^2}{\varepsilon_0} \overline{P}^* = 0 \tag{12}$$

The system (12) presents many asymptotics constraints. Tacking into account the *"Least Degeneration Principle"*, we allow us to formalize the above constraints by the subsequent system :

$$\varepsilon^2 \equiv \varepsilon_0 \tag{13}$$

the relationship seem the conection between the fluid and tube properties.

Now, the equation (12) is linearized, namely:

$$\overline{P}^* = \varepsilon_0 \ \tilde{P}^* = \varepsilon^2 \ \tilde{P}^*; \qquad \overline{\mathrm{R}}=1+\varepsilon_0 \tilde{R} = 1+\varepsilon^2 \tilde{R} \tag{14}$$

So, at order 0 in $\varepsilon_0\,(\varepsilon^2)$, we obtain the following solution:

$$\tilde{R} = -\gamma_f^0 \overline{P}^* \tag{15}$$

This model describes a linear relationship between pressure and area cross, and where γ_f^0 is the proportionality factor which is a measure for the stiffness of the tube wall. This result is valid only for a polymer structure.

The kinematic condition on the boundary is defined as:

$$\tilde{U}_1 = \frac{\partial \tilde{R}}{\partial t} \quad \text{on} \quad r \cong 1 \tag{16}$$

Taking into account the above process, we may deduce the variation of the radius of our model, namely:

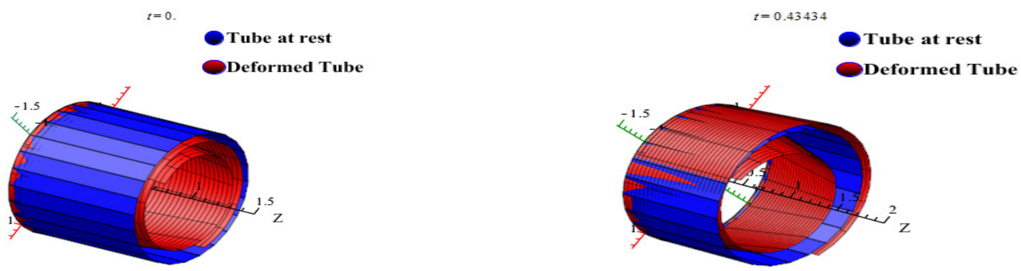
$$\tilde{R}(z,t) = -\gamma_f^o A_5 \left[\frac{\omega\sqrt{2I\omega}J_o(I\sqrt{2I\omega}/2)}{2J_1(I\sqrt{2I\omega}/2)}(-\gamma_f^o \omega\sqrt{I\omega}J_o(I\sqrt{I\omega}) + J_1(I\sqrt{I\omega}/2))z - I\sqrt{I\omega}J_o(I\sqrt{I\omega}) \right] e^{I\omega t} \tag{17}$$

Where the $J_{o,1}$ are the Bessel functions and ω is the fluid frequency. *I* is the complex number. The solution (17) shows the effects lead to interesting coupled fuid-structure problems.

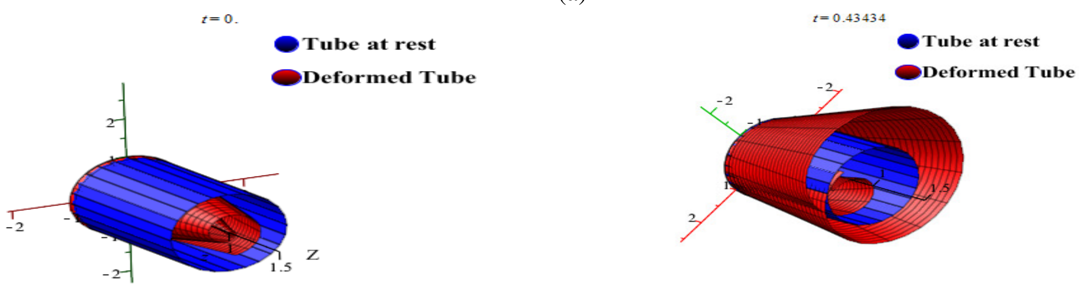
we note that the Bessel function come up in many engineering applications such as heat transfer, vibrations, stress analysis and fluid mechanics. Generaly, it put in evidence the instability mechanism.

Application

In order to validate the preceding results, we now present some examples for which we estimate the application ranges of our model. In these examples, the adimensional frequency is defined as: $1 \leq \omega \leq 40$ and the adimensional stiffness factor of the tube wall is defined as: $0 \leq \gamma_f^0 \leq 0.08$. The the aspect ratio is fixed: $\varepsilon = 0.1$. ***All the variables (r,z,t) are non-dimensional***.



(a)



(b)

Figure 2: Evolution of the elastic tube radius $\bar{R}(z,t)$ in the frequency domain model: 1≤ ω ≤30 . γ_f^0 =0.08; ε =0.1and A_5=0.01.(a) 20≤ ω ≤30 at t=0 and t=0.43434 .(b) 30≤ ω ≤40 at t=0 and t=0.43434.

The figure (2) clearly shows that the elastic tube is deformed non-axisymmetrically. The region of strongest tube deformation occurred near the tube outlet where the inside pressure is low. Consequently, the tube shape changed to nearly elliptical. These results are confirmed by the experimental results obtained by S. Nahar & al [19].

The deformation of the tube is affected by the several fundamental parameters as: γ_f^0, ε and the frequency of flow. In fact, Figure (2.a) shows that the deformation occurs on the right side of the elastic tube, compared to reference tube at rest. In the frequency range: **1≤ ω ≤30,** the deformation rate is of the order 12%. In the frequency range: **30≤ ω ≤40** (Figue(2.b), the deformation rate is of the order 80% on the right and left side. But, the deformation rate, *at the top*, is of the order 29.6%,and *in the bottom*, it is of the order 100%. We remark, in a certain range of frequency, an unstable radial position appears, which separates the region where the strongly deformation is directed towards the wall [20].

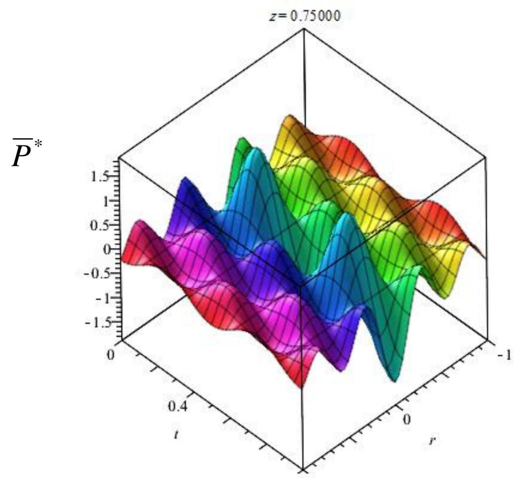


Figure 3: Evolution of the pressure $\overline{P}^*(r,z,t)$ along the z-axis.behavior at $z = 0.75$.

$-1 \le r \le 1$;$0 \le t \le 1$

Figure (3) illustrates the influences the geometrical and the deformation characteristics of the tube on the subsequent development of the initial pressure pertuebation. It appears the instabilities of axial flow and develop spontaneous oscillations of the elastic membrane[21,22] .

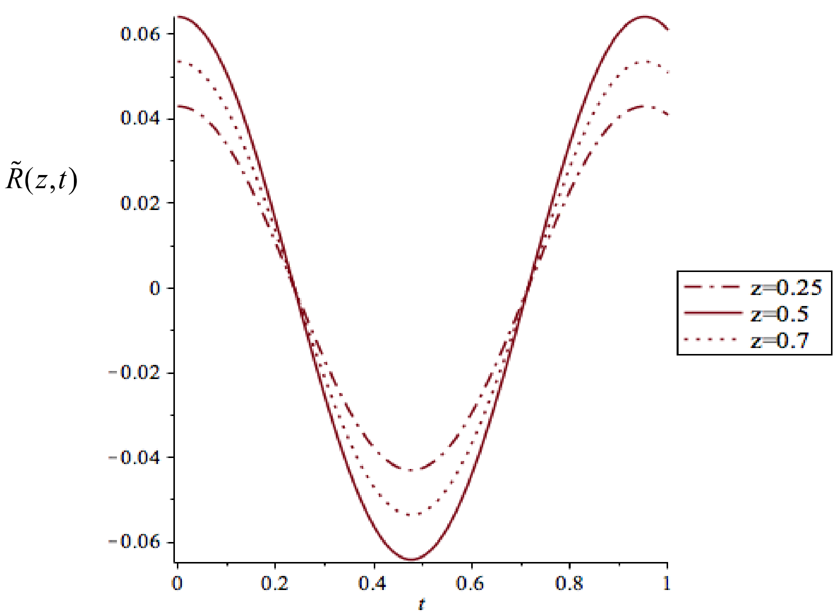


Figure 4: The variable radius $\tilde{R}(z,t)$ of cross-sections. With scaled variables:

$z = 0.25$; $z = 0.50$; $z = 0.75$ and $0 \le t \le 1$

Figure (4) shows the variation of the wall shear stress with dimensionless time along the z-axis, when the pulsatile laminaire flow has a cosine form. They appear to diminish in the negative sense at the middle of the elastic tube. If Figure (4) be compared with other works, then the validity of solution is realized [23].

Conclusion

The results obtained in the present paper globally show the importance of the vibration effect in the modeling of the behaviors and response characteristics of a three-dimensional flexible and elastic tube under laminar flow, and the power asymptotic approach for this modeling.

The several characteristics usually encountered in the fluid-structure interaction are correctly predicted by our model. When this model is applied to study of instabilities in flow past deformable solid surfaces, the matching of predictions and numerical simulations with our model remains quantatively acceptable. Moreover, they allow confirming much experimental behavior.

References

[1] Grotberg, J. B. & Jensen, O. E. 2004 Bio-fluid mechanics in flexible tubes.

[2] Squires, T. M. & Quake, S. R. 2005 Microfluidics: fluid physics at the nanoliter scale. Rev. Mod. Phys.77,977–1026.

[3] Eggert, M. D. & Kumar, S. 2004 Observations of instability, hysterisis, and oscillation in low-Reynolds number flow past polymer gels. J. ColloidInterfaceSci. 274, 234-242

[4] A.babarit et al, Modélisation numérique et expérimentale d’un système houlomoteur électro-actif déformable. 13eme Journées de l’hydrodynamique. Chatou, France, Novembre 2012.

[5] AleksanderMitin 2013, Main Gas pipelines : Fracture ResistanceAssessment of Pipes.

[6] Casson N. 1959, A flow equation for pigment oil suspensions of printing ink type. In rheology of disperse systems.Ed.Mill C.C, Pergamon London pp.84-102.

[7] Bellet D, 1973, relations entre comportements rhéologiques et échanges thermiques. Thèse de Doctorat ès-Sciences, U.P.S, Toulouse.

[8] Ly DP, Bellet D, Bousquet A et Boyer P. 1981, écoulements pulsés de fluide inélastiques en conduites tronconique ou déformable. Revue de Physique Appliquée 16 : 323-331.

[9] Rakotomalala et Bellet 1991, Ecoulements transitoires et périodiques de fluides non newtoniens en conduits tronconiques. Journal de Physique 1 :87-102.

[10] Bahrar B. 1986, Influence, sur les écoulements transitoires en conduites, des termes d’inertie de la paroi, ainsi des déformations de flexion et de cisaillement, Thèse de Doctorat de 3 cycle, INSA Lyon

[11] F.Frey et M.A Studer : analyse des structures et milieux continus : Coques , 2003.

[12] G.R.Kirchhoff :Uber das Gleichgewicht und BewegungeneinerelastischenScheibe, J. Reine u. Angew. Math 40 (1850) 51-88.

[13] a.A.JCallegari and E.L. Reiss : Nonlinear boundary value problems for the circular membrane, Arch. Rat. Mech. Anal. 31 (1968) 390-400
[14] W.T. Koiter, On the foundations of the linear theory of thin elastic shells. I, II. Nederl. Akad. Wetensch. Proc. Ser. B 73, 169{195, (1970).
[15] HsuJ.C.&CliftonR.J. (1956)Waveina thin-walled tube due to sunder release of radial ring pressure. J. Acous. Soc. Amer. 23: 563-568.
[16] Anliker M, Izaman R, IntJ.solids Structures, 1966,2,467-491
[17] Moddie T.B. &Haddow J.B. (1978) Dispersive effets in wave propagation in thin-walled elastic tubes J. Acous, Soc. Amer. 64: 522-528
[18] Serge Laroze : Mecanique des structures Tome 1 : Solides elastiques, plaques et coques, Chapitre III, Coques. Cépaduès Editions, N^{0}Editeur : 710, Septembre 2005.
[19] S. Nahar, S.A.K.Jeelani,E.J.Windhab, Influence of elastic tube deformation on flow behavior of a shear thinning fluid. Chemical Engineering Science 75 (2012) 445–455.
[20] Massimiliano M. Villone , Francesco Greco , Martien A. Hulsen , Pier Luca, Maffettone,Numerical simulations of deformable particle lateral migration in tube flow of Newtonian and viscoelastic media. Journal of Non-Newtonian Fluid Mechanics 234 (2016) 105–113.
[21] HilmiDemiray , Solitary waves in prestressed elastic tubes, Bulletin of Mathematical Biology. Vol.58,No.5,pp 939.955.1996
[22] Womersley,J.R. 1955. Oscillatory motion of a viscous liquid in a thin walled elastic tube, The linear approximation for long waves. Phil. Mag. 46, 199-219.
[23] S. Maddah, M. Navidbakhsh, and Gh. Atefi, Continuous Model for Dispersion of Discrete Blood Cells with an ALE Formulation of Pulsatile Micropolar Fluid Flow in Flexible Tube. Journal of Dispersion Science and Technology, 34:1165–1172, 2013.

Biographies

ABDESSAMAD MEHDARI is an aircraft maintenance engineer. He graduated from the Royal air force school in 2006. He is responsible for helicopters maintenance and at the same time he is Ph.Dat Research and development and mechanical multimedia modeling team department, which is a division of the Mohammadia engineering school at Mohamed V university of Rabat-Morocco.
AbdessamadMEHDARI can be reached at abdmehdari@gmail.com

MOHAMED HASNAOUI is currently a full mechanics professor at the Royal air force school of Marrakech. He is the head of Structure & Material department.
Dr.MohamedHASNAOUI can be reached at hasnaouimohammed@hotmail.com

MOHAMED AGOUZOUL is a full mechanics professor at the Mohammadia engineering school of Rabat. He is the head of Research and development and mechanical multimedia modeling team department at Mohamed V university of Rabat.
Dr. Mohamed AGOUZOUL can be reached at agouzoul@emi.ac.ma