Latency and Phase Transitions in Interconnection Networks with Unlimited Buffers

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Abstract

This paper presents theoretical and numerical results for the performance of a multiprocessor network modeled as a ring and as a toroidal square lattice of nodes with local processors that generate messages for output ports/buffers. The output buffers are assumed to have infinite capacity and service time is deterministic. Two models are considered. One assumes that every processor generates messages with rate λ per time slot and per output port/buffer. The other model considers that the generation rate of a node depends on the intensity of the flow of arriving messages. Explicit expressions for the distribution of queue lengths, the average number of messages in the buffers, the average latency and the critical network load depending on distance between the source and the destination are obtained. Simulation results show excellent agreement with theoretical predictions based on the assumption of independent queues.

1. Introduction

Modern massively parallel computers are characterized by a scalable architecture. These computers offer corresponding gains in performance as the number of processors is increased. Such computers often consist of self-contained processing nodes, with associated memory and other supporting devices. This design approach has many advantages. The repetition of identical components leads to scalability, modularity, greater reliability, and opportunities for fault tolerance. However, parallel computing in such systems requires extensive communications between otherwise independent nodes so that data and instructions are redistributed periodically to keep all processors busy performing useful tasks. Because memory is not shared between node processors, interprocessor communications are achieved by passing messages between nodes through a communications network. This network is implemented as a set of interconnected routers, each connected to its local processor. Several of the most advanced supercomputers, such as Sequala (BlueGene/Q , IBM), Titan (Cray XK7), and Trinity(Cray Xc40), have a multy-dimensional toroidal interprocessor network topology. This implementation of a network reduces the path length between nodes and simplifies routing algorithms for static or dynamic routing.
Many papers (see, e.g., [1, 20]) have been devoted to analyzing computer communication networks as networks of queues. The first most important result was obtained by Jackson. In

[3], published in 1963, he proved that for an open network of single-server queues with exponential arrival/departure rates "the equilibrium joint probability distribution of queue lengths is identical with what would be obtained by pretending that each individual service center is a separate queuing system independent of the others" (p.132). Subsequently, Gordon and Newell [4], and Buzen [5] have shown that the state distribution for the M=M=m queuing network has a product form for the first-come-first-served (FCFS) queuing discipline. (The M=M=m queuing network consists of nodes that have a Poisson flow of incoming messages, exponential distribution of service time, and m servers.) The Baskett, Chandy, Muntz, and Palacios (BCMP) theorem [6] extends this property of the state distribution for cases where the service rate is not necessarily exponential, but has a distribution with a rational Laplace transform, and the queuing discipline is one of the following four cases: FCFS, processor sharing (PS), infinite server (IS), or last-comefirst-served (LCFS). For FCFS, the service time distribution must be a negative exponential. Subsequently [7-9], three other classes of networks with exponential service times have been shown to have product form distributions. Networks with this property have been analyzed further in [10-14]. Recently, efforts of many researches focused on critical phenomena in computer communication networks [21-31]. Congestion and other phase transitions were observed and analogies with statistical mechanics were considered.

This paper presents a model of a multiprocessor network in the form of a ring and a 2-dimensional toroidal (wrapped-around) square lattice. The operation of the network is presented as a sequence of discrete time intervals (clock cycles). The specific features of the model are different from those considered in the literature. In particular, the service time is deterministic and, hence, does not have a rational Laplace transform. We obtain theoretical and numerical (simulation) results for the performance of the model. The theory is based on the assumption of independent queues. The distributions of the number of messages in each queue and of the latency (the time elapsed between message generation and its arrival to the destination), as well as the average size of the queue and the expected value of the latency as functions of the network load are obtained. The results show remarkably close agreement between the theory and simulation that demonstrates the validity of the independent queues hypothesis.

2. The model

2.1. One-dimensional case: ring topology

Each node in our model consists of a local processor, a router, and buffers of infinite capacity. In one-dimensional case each node is connected to two neighbors: the left one and the right one, and, respectively, has two output ports/buffers. The following conventions are made in the model.

1) At any clock cycle a message intended for each of the output ports can be generated by the local processor at every node, independently of others, with a constant probability □.
2) All messages are to be sent to a destination at distance exactly l hops from the source. (The distance between neighbors is equal to one hop.)
3) Any message generated at the node or arriving from a neighboring node is place immediately in the output buffer in the direction of the shortest path to the destination.

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4) At any clock cycle, if a buffer is not empty, exactly one message is transferred to the neighboring node, and it appears at that node at the next clock cycle. Thus, the service time is equal to one clock cycle.
5) If there are more than one message in the buffer, a message to be transferred is chosen at random with equal probabilities (the so called SIRO (service-in-random-order) queueing discipline).
6) At the time (clock cycle) when a message reaches its destination, it is immediately consumed and leaves the network.

2.2. Two-dimensional case: torus

In the two-dimensional case each node has four neighbors and, correspondingly, four output ports/buffers. In addition to the conventions listed in section 2.1 the following rules are accepted.

1) At any clock cycle a message intended for each of the output ports is generated independently at every node with probability λ.
2) Each of $4l$ destinations at distance l from the source has the same probability to receive a message.
3) If there is a choice between intermediate nodes on a shortest path to the destination, each of them is chosen with probability ½.
4) Two different queueing disciplines are considered:
   SIRO and the priority discipline in which a newly generated message is sent first (new-first-order, or NFO).

It is seen that our system differs from networks for which the product form of the limiting state probabilities was proved earlier in a number of characteristics.

1. Time is discrete, and arrivals occur with specific probabilities, depending on the distance l (non-Poisson and non-binomial), as given below by (12).
2. We consider both SIRO and NFO queueing disciplines.
3. The service time is deterministic and does not belong to the class of service time distributions with rational Laplace transforms.

In general, our choice of models was dictated by two opposite considerations. On one hand, the models are supposed to reflect some important features of the real-life situation. In particular, the assumption of the deterministic service time is quite natural for homogeneous messages of the same volume. This case is interesting for being researched, since it does not satisfy the conditions for which Jackson theorem is proved. On the other hand, the model should be simple enough to allow a full-fledged theoretical analysis that would reveal fundamental properties of the communication process in the network, in the most general closed form as functions of the parameters of the process. (In particular, it was important to fix the distance between the source and destination as one of such parameters). Of couse, this task cannot be accomplished by computer simulation.

3. Theoretical analysis

3.1. The ring

We consider the time evolution of the queue at each node buffer as a Markov chain where the next state depends on the present state of the buffer, while assuming the steady- state probability distribution for states of all other buffers. This approach is similar to the "mean field theory" in statistical physics. Our goal is to obtain explicit analytical expression for the distribution of the number of messages in a buffer at the steady state (equilibrium) of the network, as it depends on the load λ, and to determine the critical value of the load that results in saturation.

Let us call the *grade* of a message the number k of hops the message has made towards the destination. For the chosen value of the parameter l, there exist messages of l different grades in the system: $k = 0,1,\ldots, l - 1$, since the messages of grade l disappear from the system. Note that a message of grade k that leaves a node appears at the next node as a message of grade k+1. The state of a buffer can be described as a vector $(n_0, n_1,\ldots, n_{l-1})$ where n_k is the number of messages of grade k. Denote the limiting (steady-state probabilities) of a state by $p(n_0, n_1,\ldots, n_{l-1})$. Denote

$$\sum_{k=0}^{l-1} n_k = n. \tag{1}$$

According to SIRO discipline, the probability that a message of grade k will be transferred is equal to n_k /n. Under the steady-state condition, at any clock cycle, the expected number of messages generated at a node should be equal to the expected number of messages of grade 0 that leave the queue. On the other hand, these messages have grade 1 when they enter the next node, and their expected number is equal to the expected number of the messages of grade 1 that leave the next node, and so on. Therefore,

$$\begin{aligned}\lambda &= \sum_{n=1}^{\infty}\sum_{n_0=0}^{n}\cdots\sum_{n_{l-1}=0}^{n}\frac{n_0}{n}p(n_0,n_1,\ldots,n_{l-1}) = \sum_{n=1}^{\infty}\sum_{n_0=0}^{n}\cdots\sum_{n_{l-1}=0}^{n}\frac{n_1}{n}p(n_0,n_1,\ldots,n_{l-1}) = \ldots \\ &= \sum_{n=1}^{\infty}\sum_{n_0=0}^{n}\cdots\sum_{n_{l-1}=0}^{n}\frac{n_{l-1}}{n}p(n_0,n_1,\ldots,n_{l-1}).\end{aligned} \tag{2}$$

Since exactly one message is transferred from any state except the zero state it follows from (2) that

$$\sum_{k=0}^{l-1}\sum_{n=1}^{\infty}\sum_{n_0=0}^{n}\cdots\sum_{n_{l-1}=0}^{n}\frac{n_k}{n}p(n_0,n_1,\ldots,n_{l-1}) = l\lambda = 1 - p(0,0,..,0). \tag{3}$$

Hence,

$$p(0,0,\ldots,0) = 1 - l\lambda . \tag{4}$$

It can be shown that the limiting probabilities of all states with the same total number n of messages of all grades are equal:

$$p(n_0, n_1, .., n_{l-1}) = \frac{m!(l-1)!}{(m+l-1)!} P(n), \tag{5}$$

where n is given by (1), $\frac{(m+l-1)!}{m!(l-1)!}$ is the number of different states with n messages, and $P(n)$ is the total probability of all states with n messages.

After simplification, the balance equations for the limiting probabilities are

$$\begin{aligned}
&P(0) = (1-\lambda)(1-(l-1)\lambda)\big(P(0)+P(1)\big),\\
&P(1) = \big((1-\lambda)(l-1)\lambda + \lambda(1-(l-1)\lambda)\big)\big(P(0)+P(1)\big) + (1-\lambda)(1-(l-1)\lambda)P(2),\\
&P(2) = \lambda^2(l-1)\big(P(0)+P(1)\big) + \lambda(1-(l-1)\lambda)P(2) + (1-\lambda)(1-(l-1)\lambda)P(3),\\
&\left(\frac{1-\lambda}{l} + \frac{(l-1)(1-2\lambda)}{l}P(0)\right)P(n) = \lambda\frac{l-1}{l}(1-P(0))P(n-1) + (1-\lambda)\left(\frac{1}{l} + \frac{l-1}{l}P(0)\right)P(n+1),
\end{aligned} \tag{6}$$

for $n \geq 3$

where $P(0) = p(0,0,..,0)$ is given by (4).

Solving the system of equations (6) we obtain:

$$\begin{aligned}
&P(1) = (1-l\lambda)\left[\frac{1}{(1-\lambda)(1+\lambda-l\lambda)} - 1\right],\\
&P(n) = (1-l\lambda)\frac{\lambda^{2(n-1)}(l-1)^{n-1}}{(1-\lambda)^n(1+\lambda-l\lambda)^n}, \qquad \text{for } n \geq 2.
\end{aligned} \tag{7}$$

Hence, the distribution of the size of the queue is geometric starting with n =2.

Finally, the average number of messages in a queue is

$$\bar{n} = \sum_{n=1}^{\infty} nP(n) = \frac{\lambda^2(l-1)}{1-l\lambda} + l\lambda. \tag{8}$$

It follows that the critical load is

$$\lambda_{crit} = \frac{1}{l} \tag{10}$$

and the critical exponent at (8) and (9) is equal to 1.

3.2. The torus

It is shown that in the two-dimensional case, alike the ring case, the limiting probabilities of all states with the same total number n of messages of all grades are equal. Therefore, the equations for limiting probabilities can be written in terms of $P(n)$, where n is the total numbers of messages in the buffer. However, an important difference between 1-dim and 2-dim cases is that, while in the Markov chain for 1-dim case there are transitions from a state with n messages only to states with $n-1$, n, and $n+1$ messages $n \geq 1$), in the 2-dim case there exist transition from a state with n messages to the states with $n - 1$, n, $n + 1$, $n + 2$, and $n + 3$ messages ($n \geq 1$). Interestingly, the balance equations for $P(n)$ turn out to be the same for both SIRO and NFO.

Denote by a_i ($i = 0,1,2,3$) the probability that exactly i messages arrive to a particular buffer from neighboring nodes during one clock cycle. It follows from the description of the model given above that

$$\begin{aligned}
a_0 &= \left(1-\frac{(l-1)\lambda}{2}\right)\left(1-\frac{(l-1)\lambda}{4}\right)^2, \\
a_1 &= \frac{(l-1)\lambda}{2}\left(1-\frac{(l-1)\lambda}{2}\right)\left[\left(1-\frac{(l-1)\lambda}{2}\right)+\left(1-\frac{(l-1)\lambda}{4}\right)\right], \\
a_2 &= \frac{(l-1)^2\lambda^2}{16}\left[\left(1-\frac{(l-1)\lambda}{2}\right)+4\left(1-\frac{(l-1)\lambda}{4}\right)\right], \\
a_3 &= \frac{(l-1)^3\lambda^3}{32}.
\end{aligned} \tag{11}$$

Since a message intended for this buffer can be also generated independently with probability λ by local processor, the probability of the total number of arrivals being equal i is

$$\lambda a_{i-1} + (1-\lambda)a_i \qquad (i = 0,1,2,3,4). \tag{12}$$

Here $a_{-1} = a_4 = 0$.

Then, after simplification, the balance equations for each buffer can be written as follows:

$$\begin{aligned}
&P(0) = 1 - l\lambda, \\
&P(1) = \big((1-\lambda)a_1 + \lambda a_0\big)\big(P(0)+P(1)\big) + (1-\lambda)a_0 P(2), \\
&P(2) = \big((1-\lambda)a_2 + \lambda a_1\big)\big(P(0)+P(1)\big) + \big((1-\lambda)a_1 + \lambda a_0\big)P(2) + (1-\lambda)a_0 P(3), \\
&P(3) = \big((1-\lambda)a_3 + \lambda a_2\big)\big(P(0)+P(1)\big) + \big((1-\lambda)a_2 + \lambda a_1\big)P(2) + \big((1-\lambda)a_1 + \lambda a_0\big)P(3) \\
&\qquad + (1-\lambda)a_0 P(4), \\
&P(4) = \lambda a_3\big(P(0)+P(1)\big) + \big((1-\lambda)a_3 + \lambda a_2\big)P(2) + \big((1-\lambda)a_2 + \lambda a_1\big)P(3) \\
&\qquad + \big((1-\lambda)a_1 + \lambda a_0\big)P(4) + (1-\lambda)a_0 P(5), \\
&(1-\lambda)a_0 P(n+1) = \lambda a_3 P(n-2) + (\lambda a_2 + a_3)P(n-1) + (\lambda a_1 + a_2 + a_3)P(n), \quad (\mathrm{n} \geq 5).
\end{aligned} \tag{13}$$

The characteristic equation is cubic:

$$(1-\lambda)a_0 x^3 - (\lambda a_1 + a_2 + a_3)x^2 - (\lambda a_2 + a_3)x - \lambda a_3 = 0. \tag{14}$$

The general solution of system (13) has a form

$$P(n) = Ax_1^n + Bx_2^n + B^* x_3^n, \quad \mathrm{n} \geq 5, \tag{15}$$

where A, B, and B^* are functions of λ and l; A is real, while B and B^* are complex conjugate; x_1 is the real root of equation (15), and x_2, x_3 are two complex conjugate roots of the characteristic equation. In particular,

$$P(1) = (1 - l\lambda)\left(\frac{32}{(1-\lambda)(2-(l-1)\lambda)(4-(l-1)\lambda)^2} - 1\right).$$

Explicit expressions for larger n are too complex to be written here. However, with some technical contrivances, we have been able to obtain rather simple explicit expressions for the average number $\bar{n}$ of messages in the buffer, and the average latency τ:

$$\bar{n} = \frac{\lambda^2(l-1)(11+5l)}{16(1-l\lambda)} + l\lambda, \tag{16}$$

$$\tau = \frac{\lambda(l-1)(11+5l)}{16(1-l\lambda)} + l. \tag{17}$$

Again, the saturation point is

$$\lambda_{crit} = \frac{1}{l} \tag{18}$$

and the critical exponent is equal to 1.

4. Simulation

4.1. The ring

The simulations have been done for rings of length 8 and 16 for several values of l starting with $l = 2$. After achieving the steady-state regime, the simulation was run for about 50,000 clock cycles at 64 and 256 nodes.

The probability distribution of the number of messages (queue length) n for $l = 2$ and near-critical message generation rate is shown in Figure 1. The inverse value $1/\bar{n}$ of the average number of messages as a function of the load λ is shown in Figure 2. Figure 3 shows he inverse values of the latency $1/\tau$.

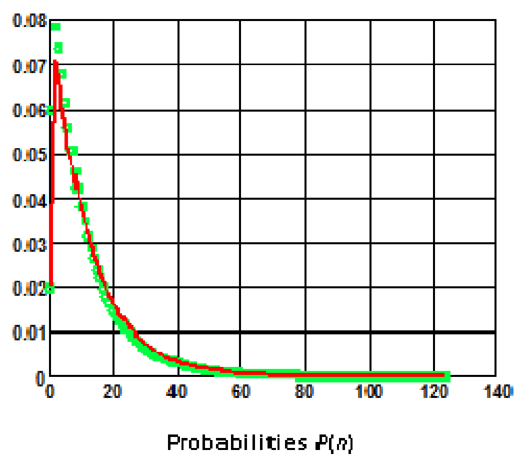


Figure 1. Probability distribution of the queue length; $l = 2$, $\lambda = 0.45$, ring length: 8 nodes. Solid line: predicted theoretical values; circles: numerical simulation resul.

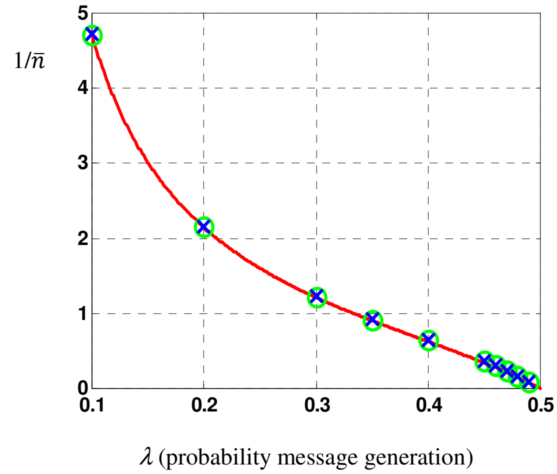


Figure 2. The inverse length of the queue for l=2. Solid line: theoretical values given by (8); numerical simulation results for ring length 8 and 16 are shown by crosses and circles, respectively.

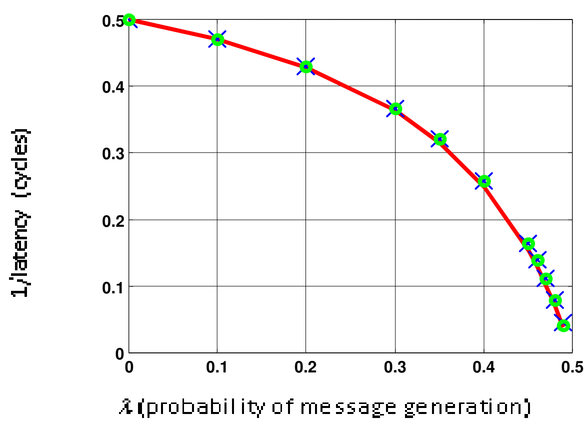
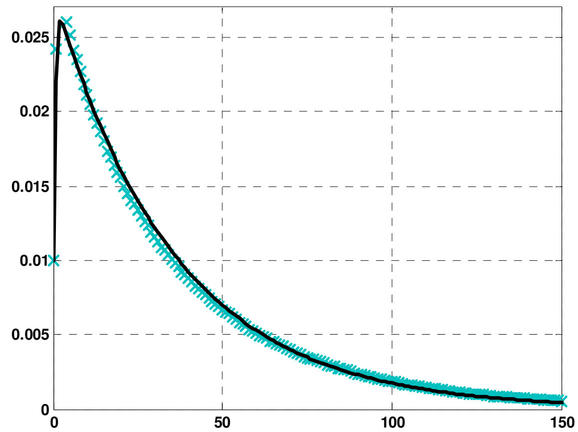


Figure 3. The inverse latency as a function the network load, l=2. Solid line: theoretical values given by (9); numerical simulation results for ring lengths 8 and 16 are shown by crosses and circles, respectively.

4.2. The torus

The simulations have been done for 16×16 toroidal square lattice for distances $l = 2$ and $l = 5$. The total number of 256×4 buffers have been observed at 15,000 points in time with intervals starting with 10 and up to 400 clock cycles to guarantee the independence of the samples. Sufficient time was allowed for the network to come to the steady-state regime before samples were taken.

The probability distribution of the number of messages $\bar{n}$ in one buffer is given in Figure 4.



Probabilities $P(n)$

Figure 4. Probability distribution of the queue length; $l = 5$, $\lambda = 0.199$, torus size: 256 nodes. Solid line: predicted theoretical values; crosses: numerical simulation results.

Figure 5 shows the inverse value of the average number of messages $1/\bar{n}$, as a function of the load λ. The standard deviations of the mean values have been calculated, but because of the large samplings size, they are too small to be shown in the plot.

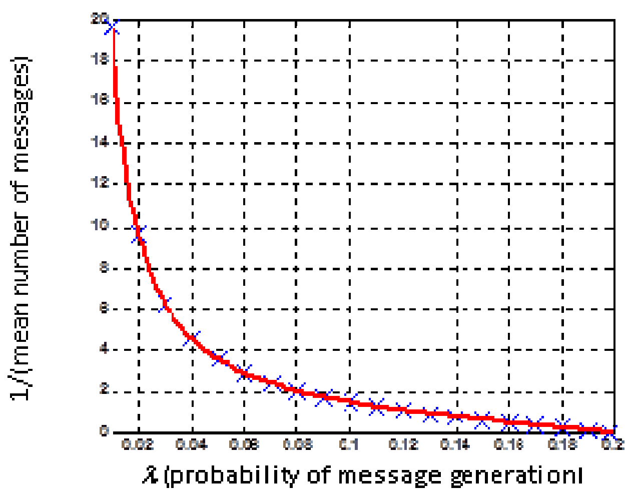


Figure 5. The inverse length of the queue for l=5. Solid line: theoretical values given by (16); numerical simulation results for torus of 256 nodes are shown by crosses.

Figure 6 shows the inverse values of the latency $1/\tau$.

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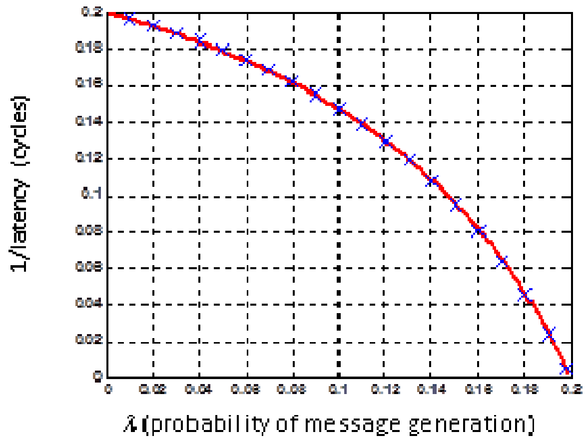

Figure 6. The inverse average latency as a function of load λ for l=5. Solid line: theoretical values given by (17); numerical simulation results for a 16×16 torus are shown by crosses.

5. Discussion of results

The obtained results show an excellent agreement between the theory and numerical experiments. Thus, the presented theoretical model of the network as a system of independent queues provides a very accurate prediction of the network behavior. This fact is remarkable, since, as pointed out in Sec. 2, our system have properties different from those for which the product form of the state probability distribution has been proved.

The saturation phenomenon in 1-dim and 2-dim interconnection networks is shown to be a phase transition with a critical exponent equal to 1. From the standpoint of statistical physics, this means that the mean field theory is exact for the types of networks considered. In contract with physical systems where critical phenomena are observed only in the "thermodynamic limit", the saturation in networks occurs for finite systems and, moreover, the characteristics of the network performance seemingly do not depend on the size the system provided that the distance l is smaller than the maximum distance between nodes. It is seen that in both 1-dim and 2-dim cases the pase transition is continuous (of the second order). The critical load is inversely proportional to the distance between the source and the destination. This fact is not surprising, since the utilization of every link in the network (the fraction of time when the link is busy) is

$$\rho = 1 - P(0) = l\lambda. \qquad (19)$$

Also, the analysis shows that the choice of the queueing discipline has no effect on the network behavior.

6. Conclusion and future work

The results of the paper demonstrate the existence of a deep analogy between phenomena in communication networks and in systems studied by statistical physics. It opens a prospect for the development of a consistent physical theory of computer communication systems that would be able to employ the powerful and well-developed apparatus of statistical physics for the analysis and control of the processes in communication networks. The other possible extension of our research is application of a similar approach to other network topologies, sets of parameters, queueing disciplines, etc.

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